NASA Explorer Schools Pre-Algebra Unit

Lesson 2 Teacher Guide

Solar System Math

Comparing Mass, Gravity, Composition, & Density

http://quest.nasa.gov/vft/#wtd

EG-2007-02-201-ARC

Table of Contents

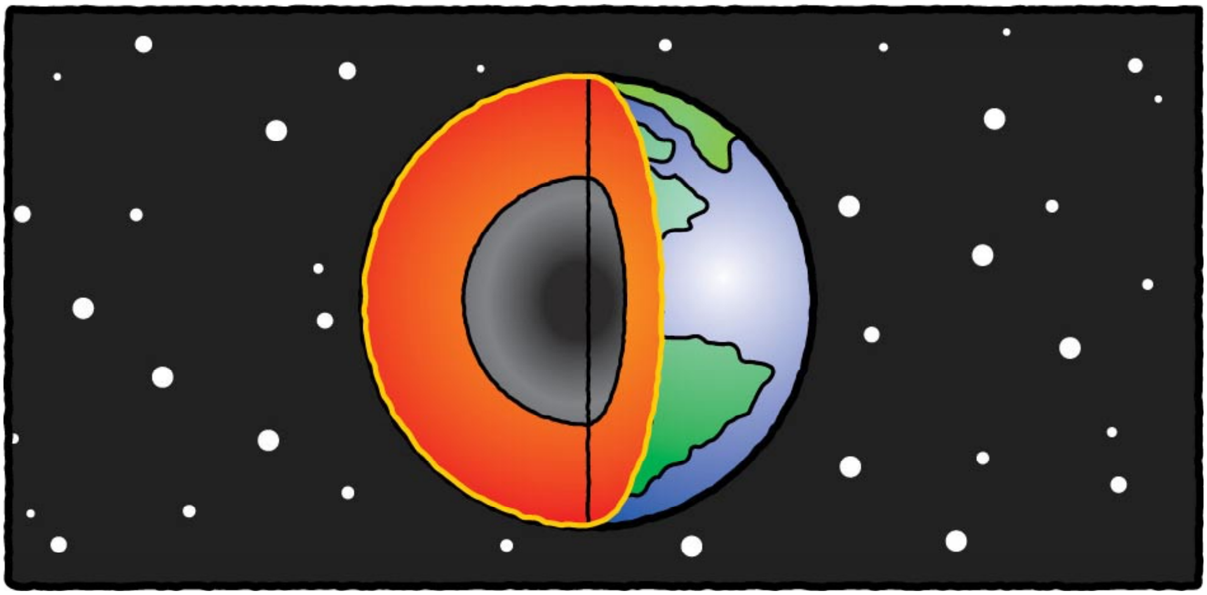
LESSON 2 FOUNDATION & OVERVIEW

LESSON 2 COMPONENTS

NOTE: A "session" is considered to be one 40-50 minute class period.

Solar System Math

Comparing Mass, Gravity, Composition, & Density



Lesson 2

What interval of values for the mass of a planet or moon has surface conditions suitable for human exploration?

Introduction

This lesson will introduce students to the differences between weight and mass. Students will gather information on the planets and moons in our solar system and use the data to create a scale model for our system in terms of mass and volume. Using fractions, decimals, percents, and number lines, students will compare the gravity and density of the planets and moons. Students will use the density of Earth materials to determine a suitable range of values for the densities of planets and moons for humans to visit. Students will use the scale model and their numerical analysis on it to further refine their ideas as to where in our solar system humans should be sent to explore.

Lesson 2 – OBJECTIVES, SKILLS, & CONCEPTS

Main Concept

The mass of a planetary body affects the surface conditions.

Instructional Objectives

During this lesson, students will:

- Create a mass/volume/density scale model of the solar system.
- Compare planet and moon masses to Earth's mass using fractions, decimals, and percents.
- Identify the interval of values for mass that will allow a planet to have a surface that can be visited by humans.
- Graph the bodies in the solar system whose interval of values for mass is suitable for visits by humans and those that are not.

NATIONAL EDUCATION STANDARDS		
Fully Met	Partially Met	Addressed
NCTM (3-5) Geometry 4.4 (3-5) Data Analysis and Probability #1.3 (6-8) Number and Operations #1.2 (6-8) Number and Operations #1.4 (6-8) Measurement #2.5 Problem Solving #1 Problem Solving #2 Communication #2 Connections #3	NCTM (3-5) Measurement #1.1 (3-5) Measurement #2.2 (6-8) Data Analysis and Probability #1.1 (6-8) Data Analysis and Probability #1.2	NCTM (3-5) Measurement #1.2

Major Focus Skills

Math topics covered in this lesson:

- Data representation (graphing)
- Comparing and ordering fractions, percents, and decimals
- Solving problems involving scale, ratio, and proportion
- Converting ratios, fractions, decimals, and percents
- Measuring circumference
- Estimating and rounding
- Finding patterns and relationships
- Calculating density using mass and volume

Major Focus Concepts

Math

- An interval on a number line can be used to specify a set of numbers that satisfy certain criteria.
- Fractions, decimals, and percents are all used to represent numbers and to describe relationships among numbers.
- Fractions, decimals, percents, and whole numbers can be placed on a number line to represent their relative values.
- A great circle is a circle on the surface of a sphere that divides the sphere into two equal hemispheres.
- The circumference of a sphere is equal to the circumference of a great circle on the sphere.
- Estimating the value of π by calculating the ratio of the circumference to the diameter of a circle.

Science

- In our solar system, more massive planets tend to be less dense, have more gravity, and are composed primarily of gas.
- In our solar system, less massive planets tend to be more dense, have less gravity, and are composed primarily of rocky materials.

- Humans cannot visit gas giants because they lack solid surfaces on which to stand, and humans could not withstand their large gravitational forces.
- It is thought that bodies with masses of up to approximately 14.9 Earth masses are rocky.
- Density is the measurement of mass per unit volume.

Prerequisite Skills and Concepts

Math Skills

- Representing and comparing decimals, whole numbers, and fractions
- Rounding decimals
- Basic calculator functions
- Concepts of and relationships among decimals, fractions, and percents
- Conversion between decimals, fractions, and percents
- Finding equivalent fractions
- Calculating scale and working with ratios and proportions (See Lesson 1)
- Understanding ratios (a comparison between two values written as a fraction or with a colon)

Math Concepts

- Diameter is the length of a line segment that passes through the center of a circle (sphere) with each end of the segment lying on the circle (sphere).
- Circumference is the perimeter of a circle, i.e., the distance around a circle.

Science Concepts

- Mass is the amount of matter in an object.
- Gravity is a force.
- Weight is a measurement of the force of gravity on the mass of an object.
- The Earth is the third planet from the Sun in a system that includes Earth's Moon, the Sun, eight other planets and their moons, and smaller objects such as asteroids and comets—all of which vary greatly in terms of their size and distance from the Sun. (See Lesson 1)

SW = student workbook TG = teacher guide EG = educator guide

Lesson 2 – PRE-LESSON ACTIVITY

- **Estimated Time:** 1 session, 40 minutes
- **Materials:**
  - 3 identical opaque containers (i.e. ice cream cartons)
  - Water
  - Sand (or alternative substance)
  - Scale or triple beam balance
  - Pre-Lesson Activity worksheet (SW pp.2-3)

Misconception Alert!

Mass and weight are directly related. Weight measures gravity's effect on mass. Because these two concepts are so closely related, some people often mistake them as being the same. Since different planets have different gravitational forces, it is important that students understand the difference between the two terms. It is also important that students understand that the composition of an object is directly related to its mass, and that two objects of the same size and the same shape typically have different masses if their compositions are different. The following activity addresses these topics.

Before the lesson, prepare the materials for the students by obtaining three identical opaque containers that do NOT allow you to see the contents. Fill one container with water. Fill another with sand or an alternative substance. Leave the other full of air. It would be best to seal the containers so that the sand and water do not spill. For discussion purposes, label the containers A B C.

Note: Students will probably pick up on the difference between the containers when you set them down. If possible, have the containers arranged in front of the class BEFORE the students arrive.

First, present the three containers to the class and, using the **Pre-Lesson Activity worksheet** (SW pp.2-3), have the students study the containers and make key observations using the prompts in questions 1-5. Next, open a discussion on the difference between *mass* and *weight* and have students draw conclusions based on questions 6-11. Finally, wrap up the activity by having students guess the contents (composition) of the three containers as directed in question 12.

Objects of the same size and shape may not have the same mass or weight based on their composition. For example, compare a marshmallow and a rock of the same size and shape. They may look similar, but they do not weigh the same and their mass is not the same because they are made of different things—they have different *compositions*.

MASS is the amount of "stuff" (MATTER) in an object.

WEIGHT is a measurement of the force of gravity on the mass of an object.

On Earth, we can use weight to see changes in our mass because the amount of gravity on Earth does not change. Yet when we weigh ourselves, we are in fact *measuring the force of gravity* on our bodies, i.e. our mass.

Because space has microgravity (very weak gravity), the WEIGHT of an object in space is significantly less than here on Earth. However, the amount of stuff that is in an object—its MASS—is not affected by gravity and therefore does not change whether it is in space or on Earth.

This same principle holds true on other planets and moons. An object's weight will vary from one planet to the next because gravitational pull varies from planet to planet. However, an object's makeup does not change from one planet to the next; therefore, its mass stays the same. As a result, objects that are nearly weightless in microgravity may still require significant effort to move (push or pull) due to their mass.

Note: Scales and balances on Earth measure mass based on Earth's gravity. If you left Earth, the amount of matter would not change, but you would need a different method of measuring mass because the scales would no longer be accurate. In orbit, astronauts use an inertial balance to measure mass. An inertial balance moves an object back and forth to measure how many times it will oscillate in a given time. This tool uses Newton's first law of inertia to measure mass.

Key Observations and Conclusions!

Even though objects have the same size and shape, they may weigh different amounts and have different masses based on their composition or matter.

An object's weight will change in space and on other planets, but its mass will remain unchanged.

A heavier (more massive) object is more difficult to move than a lighter (less massive) object. This is true on Earth as well as in the microgravity environment of space. For example, containers A, B, and C would be nearly weightless in microgravity; however, it would still be easier to move the less massive container and harder to move the more massive container.

In Lesson 2, students will look at the mass of the planets and investigate how this is related to the composition of the planets.

SW = student workbook TG = teacher guide EG = educator guide

Lesson 2 – ENGAGE

• **Estimated Time:** 2 sessions, 40 minutes each

• **Materials:**

- Planetary Mass and Human Exploration worksheet (SW p.4)
- Minimum and Maximum Values worksheet (SW p.5)
- Circles and Spheres worksheet (SW p.6)
- Transparency #1: Circles and Spheres (TG p.14)
- (Optional) Globe
- (Optional) Basketballs and other playground balls or spherical objects
- (Optional) String or tape measures
- Volume of a Sphere: A Hands-On "Proof" (TG pp.16-19)
- Transparency #2: A Cubic Kilometer (TG p.20)

1. MASS AND COMPOSITION

Students observed the vast range of sizes of the objects in the solar system in Lesson 1. What those planets are made of—their composition—will greatly affect whether humans can visit them or not.

Remind the students of their final goal:

To determine where in the solar system NASA should send humans.

In Lesson 1, students made a scale model of the solar system to compare the sizes and distances of the planets. Activate students' prior knowledge and review what they learned in Lesson 1 and in the Lesson 2 Pre-Lesson Activity by asking the following questions:

Review conclusions from Lesson 1:

- Based on your experience with a scale model of our solar system, where do you think we should send humans in our solar system? Why?
- What else do you need to know about the planets before you make your final decision?

Review information from Lesson 1 and the Lesson 2 Pre-Lesson Activity:

- What are the largest planets in our solar system? (*Outer planets / gas giants*)
- What are the smallest planets in our solar system? (*Inner / rocky planets*)
- Which planets are closer to the Sun: smaller or larger planets? (*Smaller planets*)
- What is the difference between mass and weight? (*Mass is a measurement of how much matter an object contains; weight is a measurement of gravity's pull on that mass.*)

Explain to the students that we want to consider as many options within our solar system as possible. In addition to the nine planets, there are many moons that may be appropriate places to visit; therefore, we will also consider:

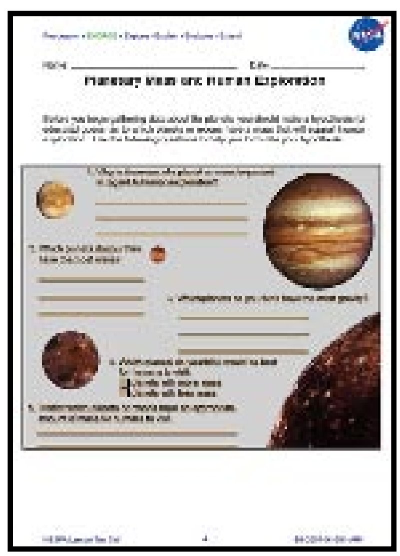
- Earth's Moon (which humans have already visited)
- Titan (a moon of Saturn)
- Triton (a moon of Neptune)
- and Io and Europa (moons of Jupiter)

Students will learn more about these moons in the EXPLORE section of this lesson when they gather data from *What's the Difference*.

In this lesson, students will explore the following scientific question:

What range of values for the mass of planets and moons would be acceptable for humans to visit?

Use the **Planetary Mass and Human Exploration worksheet** (SW p.4) to guide the class in a discussion about why the mass of a planet or moon is important in regard to human exploration.



2. MINIMUM AND MAXIMUM VALUES

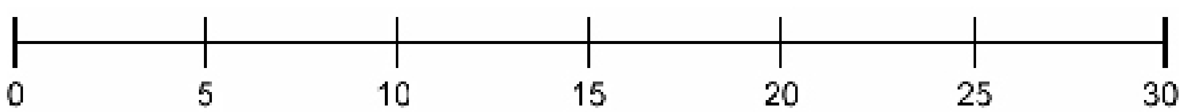
Students will be determining an acceptable *minimum and maximum mass* of planets and moons that humans can visit. In order to do this, they need to be familiar with the mathematical concept of minimum and maximum, which can be introduced by relating the concept to a real-life situation. The exercise on the **Minimum and Maximum Values worksheet** (SW p.5) can be done in groups or as a whole class.

This presents a good opportunity to show the mathematical symbolism for expressing an interval of values. It uses the symbol ≤ which stands for the words “less than or equal”. In regard to the sample response on page 5 of the Answer Key, if the symbol “x” stands for “the temperature (ºF) in Chicago on a given day in January,” then this can be expressed symbolically as:

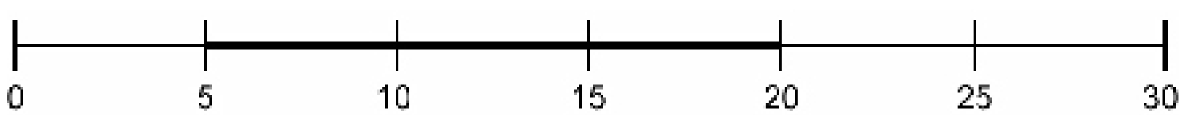
$$5 \leq x \leq 20$$

Another way to represent an interval is to use a **number line**. Mark off a number line on the board or on large graph paper with a minimum value of 0 and a maximum value of 30. As you create the number line, explain to the students that a number line has a uniform scale, i.e., the distance between each pair of consecutive whole numbers remains the same. (The distance between 4 and 5 is the same as the distance between 9 and 10, which is the same as the distance between 29 and 30, etc.)

The sample number line on page 5 of the Answer Key goes from 0 to 30, and the segments are marked in multiples of 5. (You can customize your number line to your students’ responses.)



To show the temperature interval, students can highlight the portion of the number line between, and including, 5 and 20.

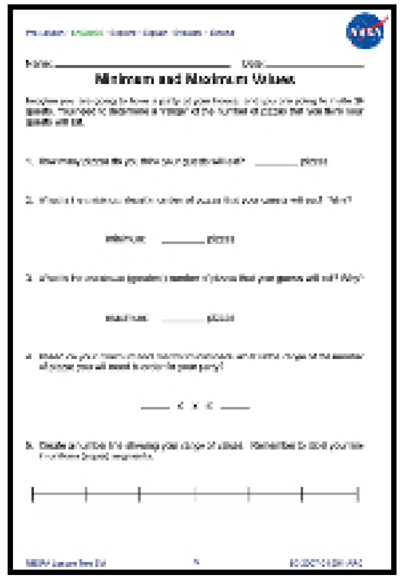


The interval includes any number between 5 and 20, including 5 and 20. Any other numbers, such as 3 or 35, would fall outside of this interval.

Other examples of when you might use an interval of values include:

- The height of a roller coaster track
- The depth of water in a reservoir in one season
- The distances people travel to school or to work

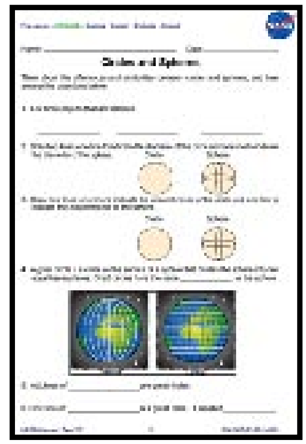
Later in this lesson, students will use a number line to describe the planets that would be acceptable for humans to visit.


Minimum and Maximum Values

3. GREAT CIRCLES

Students will need to be able to measure the circumference of a great circle on the models they will be building in the EXPLORE portion of this lesson; therefore, is important that students are familiar with the geometry of spheres.

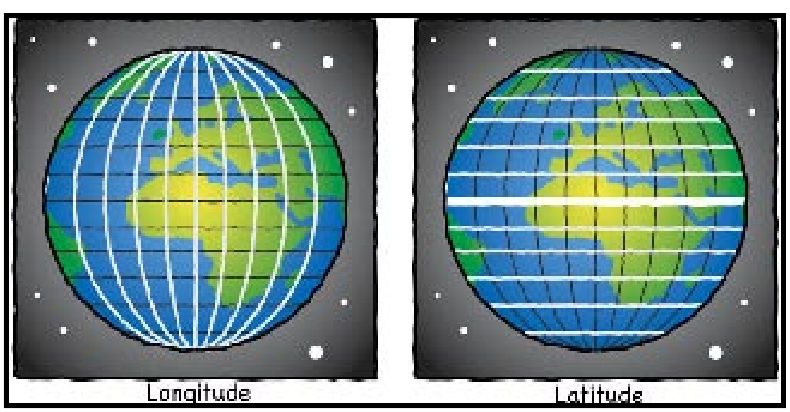
When working with more abstract concepts, it is important to show the students as many concrete examples as possible. Begin by hosting a class discussion about the differences and similarities between circles and spheres. Next, using the **Circles and Spheres worksheet** (SW p.6) and the **Circles and Spheres transparency** (TG p.14), discuss the following:



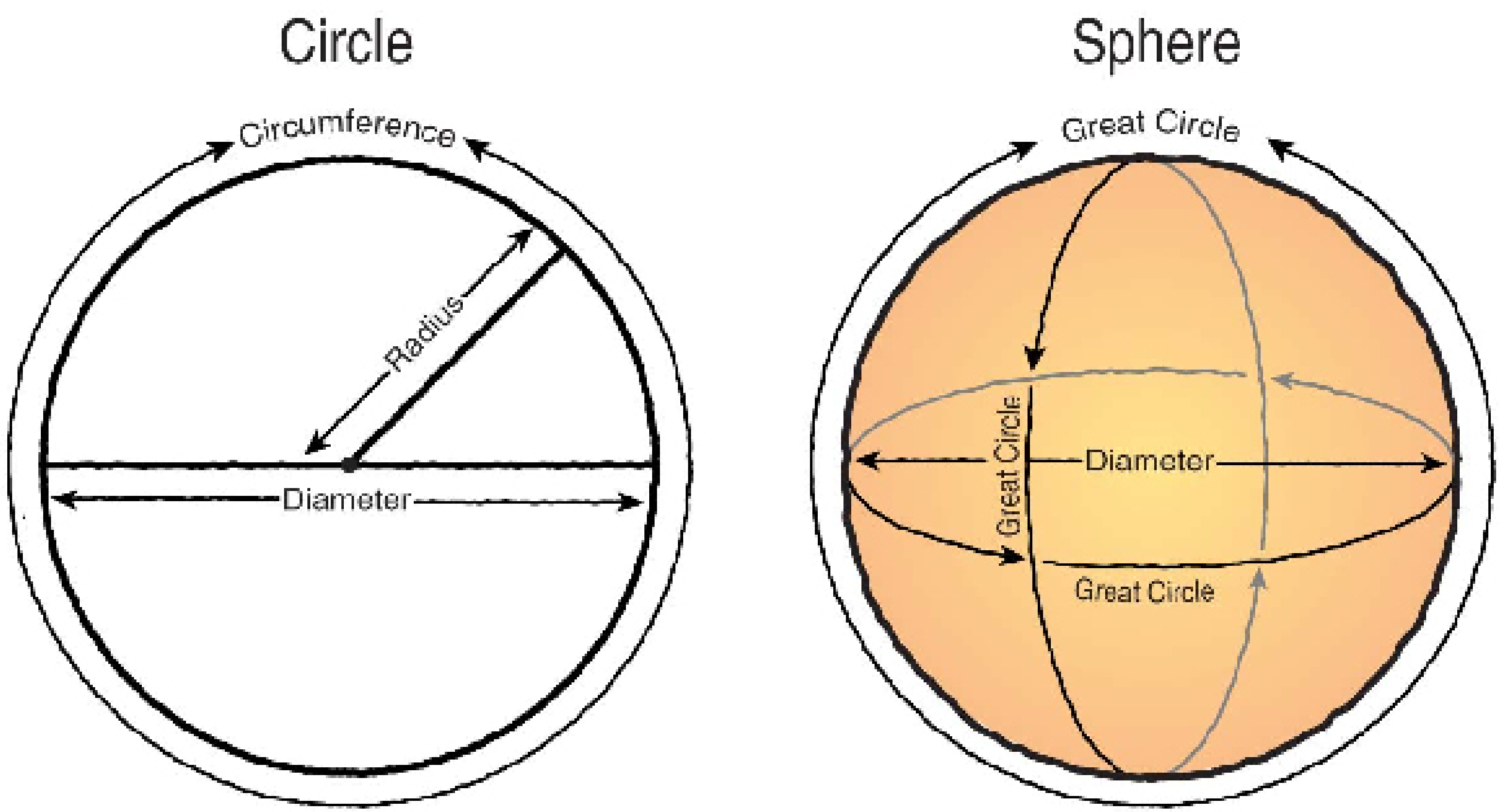
- What are some spherical objects?
- What is the diameter of a circle? (The length of a line segment that passes through the center of a circle with each end of the segment lying on the circle)
- What is the circumference of a circle? (The perimeter of a circle, i.e., the distance around the circle)
- What is the diameter of a sphere? (The length of a line segment that passes through the center of a sphere with each end of the segment lying on the sphere)
- What is the circumference of a sphere? (The perimeter of a sphere, i.e., the distance around the sphere. *The circumference of a sphere is equal to the circumference of a great circle on a sphere.*)

Misconception Alert!

A great circle is a circle on the surface of a sphere that divides the sphere into two equal hemispheres. Great circles have the same circumference as the sphere. On Earth, ALL lines of longitude are great circles because each line of longitude circles the Earth and divides the Earth into two equal hemispheres. However, only one line of latitude is a great circle—the equator. No other lines of latitude are great circles because they do *not* divide the Earth into two equal hemispheres; their circumferences are too small.



Transparency #1: Circles and Spheres



4. VOLUME

Students should understand the concept of volume as it is related to density, but they will not need to calculate volume in this lesson. For the purpose of this lesson, students need to understand that *volume is a measurement of the capacity of an object*—how much something will hold.

Discuss which units are used to measure volume. Students should realize that there are both metric and standard units, and that different scales are used depending on the volume size. There are also different units for measuring volume of space of large objects versus dry and liquid substances, such as foods or other store goods. Furthermore volume is measured in three dimensions, which is why space is measured in cubic meters or cubic feet.


Volume

Note: If you choose to do extended activities with students once they understand volume, the formula for the volume V of a sphere (such as a planet) is:

$$V = \frac{4 \pi r^3}{3}$$

where r is the radius of the sphere.

Before introducing this or any formula, it is important to guide students in understanding the concepts behind the formula. Help students to develop the formula for themselves by looking for patterns and relationships as described in **Volume of A Sphere: A Hands-On "Proof"** (TG pp.16-19). By doing this, students will retain and apply the concept to other situations rather than simply memorize the formula.

Ask students what unit they would use to measure the volume of *planets*? (kilometers cubed)

Volume units for large objects such as planets are usually given in kilometers cubed or miles cubed because cubic meters or cubic feet are very small relative to a planet's size. As the students saw in the scale model of the solar system in Lesson 1, the volume of each planet is very large. For example, the volume of Pluto (the smallest planet) is 7,150,000,000 km^3 (about 7 billion km^3). If planetary volume were measured in cubic meters or cubic feet, then each planet would require many such units and the students would have to work with very large numbers. Help students to connect this to their knowledge of astronomical units (AU) that they learned in Lesson 1.

A cubed kilometer can be represented by a cube one kilometer high, one kilometer wide, and one kilometer deep. **Transparency #2: A Cubic Kilometer** (TG p.20) illustrates this.

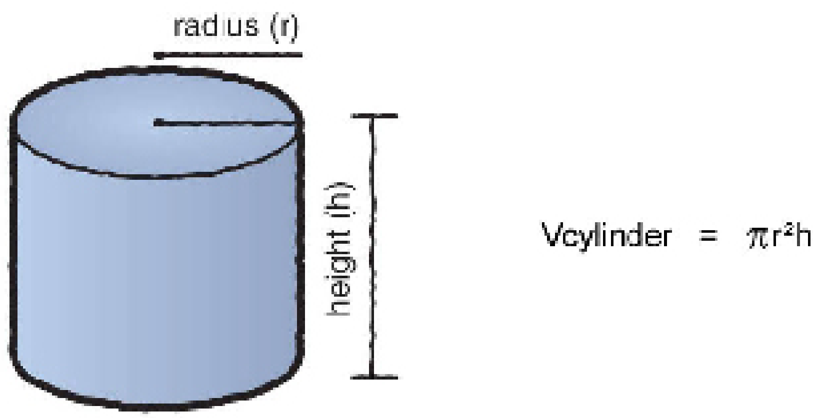
Volume of a Sphere: A Hands-On "Proof"

Students should be familiar with three-dimensional geometric shapes. In this geometric idea for a proof, they will be using cylinders, cones, and spheres to determine the *volume* of a sphere (how much the sphere can hold).

Volume of a Cylinder

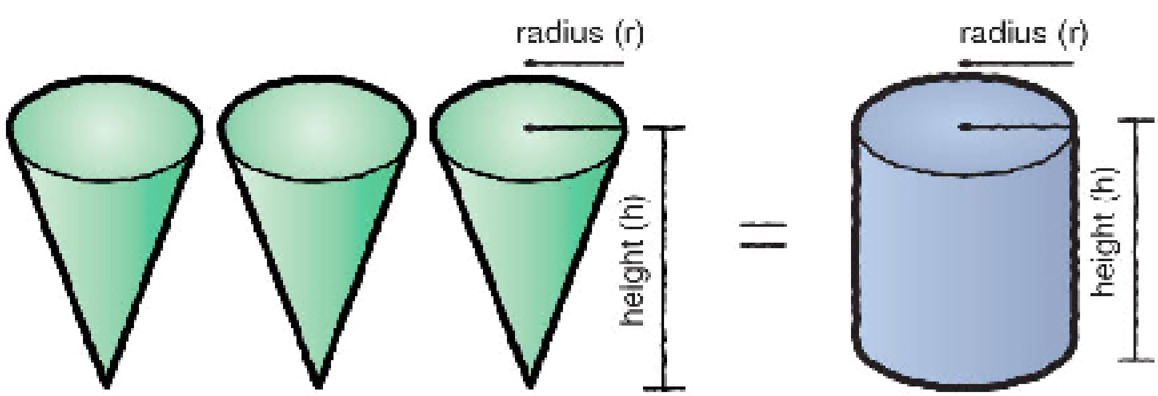
You can imagine that a cylinder is made up of many circles of radius (r), stacked up to height (h).

The volume (V) of a cylinder is the area of the circle of radius (r) times the height (h) of the cylinder. The area of the circle is πr^2.

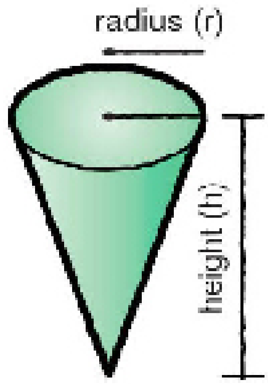


Volume of a Cone

If you had a cone with the same height (h) and the same radius (r) as the cylinder, it would take three cones to fill up the cylinder. (You can try this for yourself with sand or beans.)



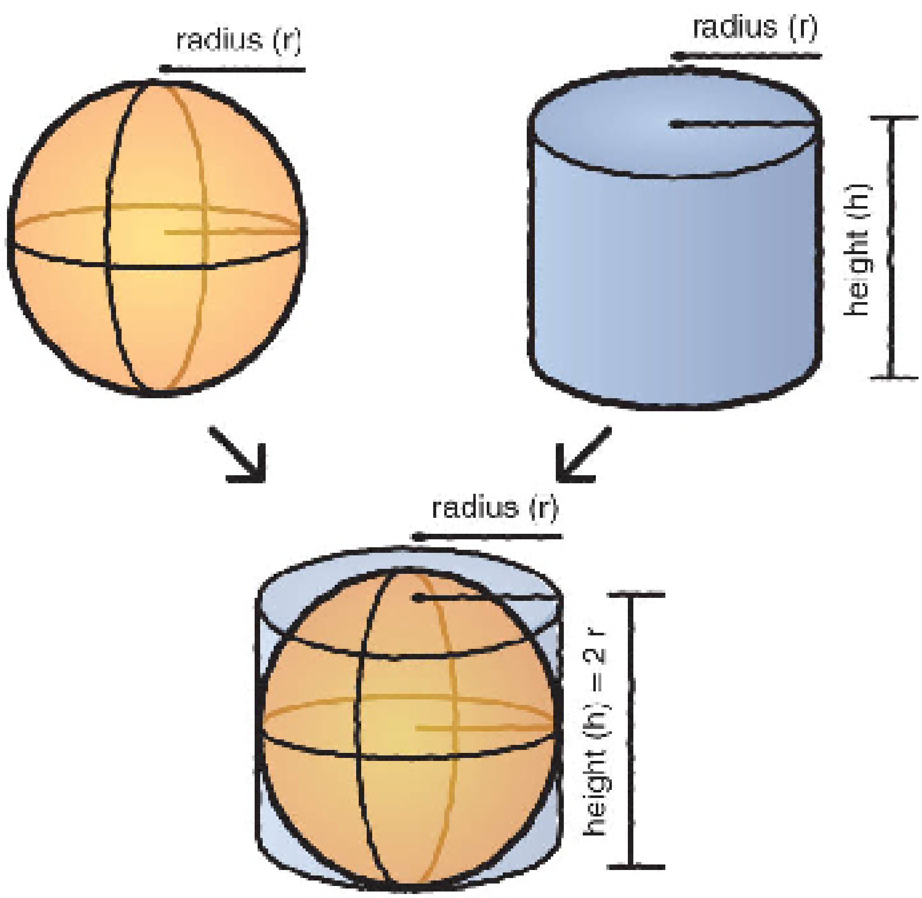
So the volume of a cone (of radius r and height h) is one-third the volume of a cylinder (of radius r and height h).



$$V_{cone} = \frac{V_{cylinder}}{3} = \frac{\pi r^2 h}{3}$$

Volume of a Sphere

If you insert a sphere with radius (r) inside a cylinder so that it fits tightly, the radius of the cylinder will be r, and the height of the cylinder will be 2r.



Even though the sphere has been placed inside the cylinder so that it fits tightly, you notice that there is still some space in the cylinder around the sphere that has not been filled.

We know the volume of the cylinder is the area of the circle of radius (r) multiplied by the height (h).

$$V_{cylinder} = \pi r^2 h$$

We know that the height (h) of the cylinder is equal to 2r.

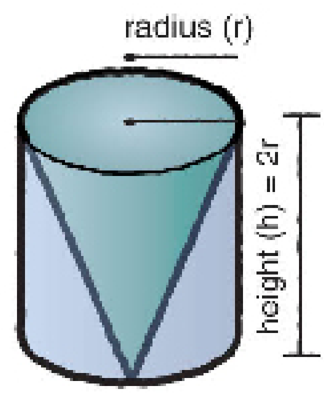
$$V_{cylinder} = \pi r^2 (2r)$$

$$= 2 \pi r^3$$

If we have a cone of radius (r) and height (2r), we can calculate its volume as follows:

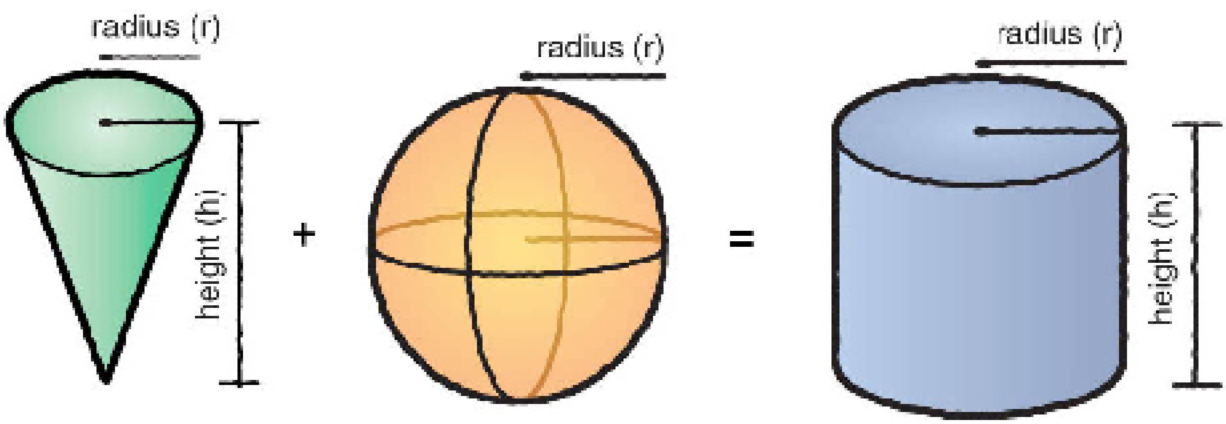
$$V_{cone} = \frac{\pi r^2 h}{3}$$

$$= \frac{2 \pi r^3}{3}$$

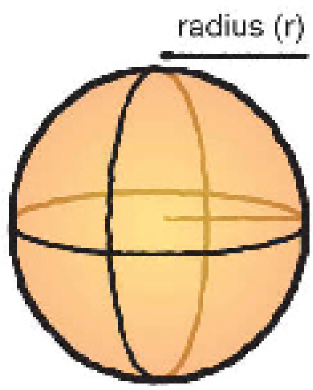


It turns out that we can fill that remaining space in the cylinder around the sphere with exactly the amount of sand that will fit into this cone. So we know that the volume of the sphere with radius (r), plus the volume of the cone with radius (r) and height (2r) is equal to the volume of the cylinder with radius (r) and height (2r).

Vcone + Vsphere = Vcylinder



We know the formulas for the volume of the cone and the volume of the cylinder. We can use these to solve for the volume of the sphere.



$$
\begin{aligned}
V_{sphere} &= V_{cylinder} - V_{cone} \\
&= 2\pi r^3 - \frac{2\pi r^3}{3} \\
&= \frac{6\pi r^3}{3} - \frac{2\pi r^3}{3} \\
&= \frac{6\pi r^3 - 2\pi r^3}{3} \\
&= \frac{4\pi r^3}{3}
\end{aligned}
$$

This is the formula for the volume of a sphere that can be rigorously derived using calculus.

This geometric idea of a “proof” that has been presented above can be tested hands-on by using a tennis ball, a cylinder, and a cone made of heavy paper.

---

5. DENSITY

In science, density is defined as mass per unit volume.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

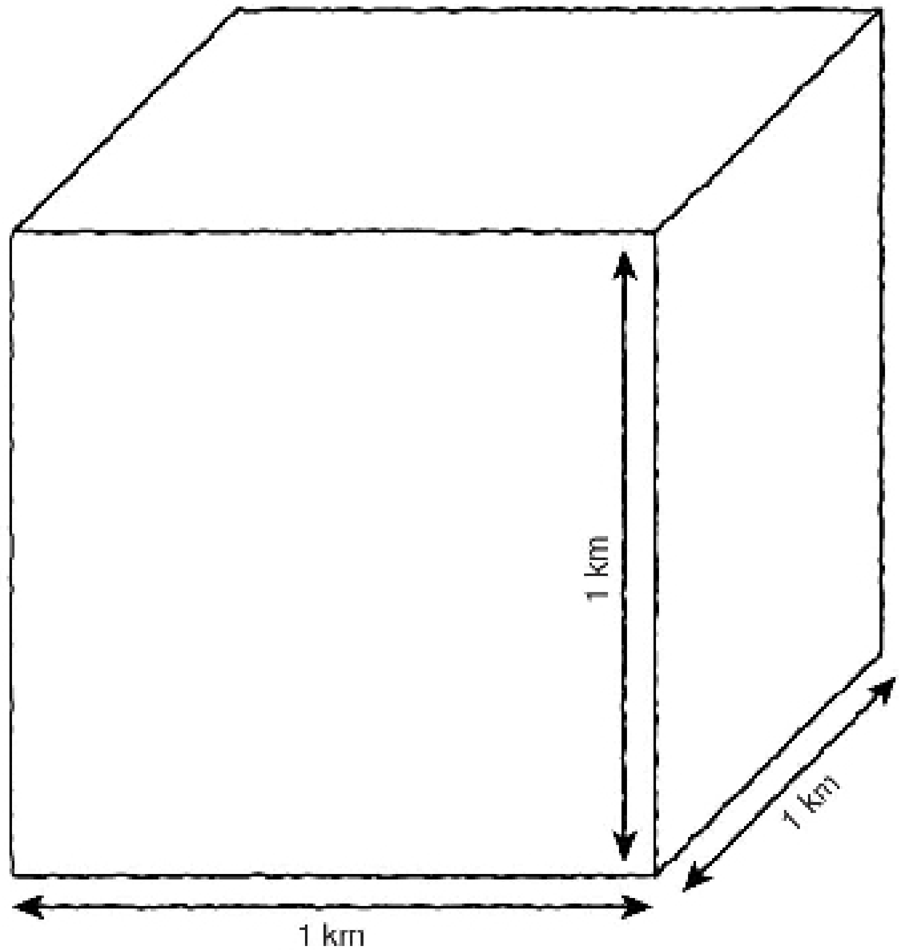
Some of the ways density can be measured are:

$$\frac{\text{grams}}{\text{meters}^3} \qquad \frac{\text{kilograms}}{\text{meters}^3} \qquad \frac{\text{kilograms}}{\text{kilometers}^3}$$

Note: Even though the composition of a planet is not homogenous, we approximate its density using this formula.

These formulas will be used on the **Volume and Density worksheet** (SW p.13) in the EXPLORE portion of this lesson. See the sample problem on **page 34** of this teacher guide.

Transparency #2: A Cubic Kilometer



Pluto, the smallest planet in our solar system, would hold about 6 billion (6,000,000,000) of these cubes!

SW = student workbook   TG = teacher guide   EG = educator guide

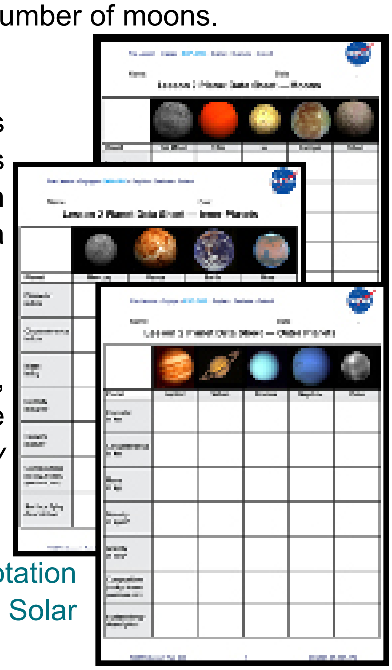
Lesson 2 – EXPLORE

- **Estimated Time:** 2 sessions, 50 minutes each
- **Materials:**
  - Lesson 2 Planet Data Sheets (SW pp.7-9)
  - Computers with *What's the Difference: Solar System* dataset
  - Calculating Mass Scale Without Scientific Notation (TG pp.25-27)
  - Calculating Mass Scale With Scientific Notation (TG pp.28-29)
  - Scale Model: Mass worksheet (SW p.10)
  - Scale Model: Circumference worksheet (SW p.11)
  - Scale Model: Observations worksheet (SW p.12)
  - Volume and Density worksheet (SW p.13)
- **Per Solar System Model:**
  - about 450 cotton balls
  - 9 plastic produce bags
  - 9 twist ties
  - Scissors, string, foil, masking tape (or index cards)
  - Markers, pens
  - (Optional) Calculators

1. DATA COLLECTION

Students will use the ***What's the Difference: Solar System*** dataset to gather data about the mass and composition of bodies in our solar system to determine *which planets and moons have an appropriate amount of mass for humans to visit.*

Ask the students what details they will need to know about the planets and moons related to mass in order to answer the scientific question. Guide their responses to appropriate factors including mass, composition, gravity, surface conditions, and number of moons.



Review with students how to use *What's the Difference*. Groups or teams of students can gather data on two planets or moons on the **Lesson 2 Planet Data Sheets** (SW pp.7-9), and then they can complete the charts as a class. (A large class chart or a transparency would be helpful to record all of the information.)

Allow students time to look at data on *What's the Difference*, especially the "surface flyby" feature. Have them compare the appearance of gas giants versus rocky planets. *What do they notice?*

Note: Mass will be given using the factor 10^{22}. This notation is explained in Section 2.1 "Calculating a Mass for the Solar System Scale Model" (TG p.23).

2. CREATING A MASS AND VOLUME SOLAR SYSTEM SCALE MODEL

Since mass, density, and volume are all rather abstract terms and the values of these measurements for the planets are extremely large, it is helpful to create a scale model to represent these values.

In this portion of the lesson, *students will create a scale model for the mass and the volume of the planets in our solar system.* The model in Lesson 1 reflected the size of the planets and their distance from the Sun. In this lesson, the model will reflect the mass of the planets and their volume. The circumference of a planet gives us the information we need in order to calculate its volume.

In Lesson 1, the challenge of making a scale model for the solar system for both size and distance was that either the planets had to be very small or the distances had to be very large. The same challenge applies here.

On their planetary data sheets, students described the composition of the planets as either rocky (or frozen) or gaseous. The students might be able to make their model using rocks but measuring the mass would be very difficult, and it is not very feasible to have them use gas in their model. Since they cannot create their model using the actual materials the planets are composed of, they will have to use a substitute. Cotton balls can be compressed to make them hard ("rocky") or fluffed up to make them more airy ("gaseous").

Cotton balls are a good compromise as to what material to use, but the model still cannot be made to the same scale for both volume and mass. In order to have the planets on the same scale for volume and mass, the planets would either have to be very small or very massive. For example, a model of Pluto on the same scale for mass and volume would be 0.8 mm wide. (Have the students look at a millimeter on a ruler and imagine making a model planet that small.) Furthermore, if the students tried to make the planets larger and on the same scale for mass and volume, then they would have to use a lot more cotton—Jupiter alone would need over 20,000 cotton balls!

> **Note:** If the students were to make a scale model for the *density* of the planets, the inner planets would need to be extremely small to make the cotton feel like rock. If the students made the scale model larger, the number of cotton balls needed to represent the mass of the planets would become unreasonably large.

Since it is not feasible to make a scale model of the planets that reflects the same scale for both the volume and the mass of the planets, **the model will reflect one scale for mass and one scale for volume.**

2.1 Calculating a Mass for the Solar System Scale Model

Students will calculate the values for the scale mass and scale volume of all of the planets and moons; however, they will only build the models of the planets.

First, students will need to calculate the scale mass for each of the planets in the model. In *What's the Difference*, the mass of the planets is given in kilograms. *Ask the students what unit they should use for mass in their model? Why would pounds or kilograms be too big?*

In order to simplify calculations, they will use 1 cotton ball to represent Earth. (1 cotton ball has a mass of approximately 0.25 grams). While the students will not be making models of the moons, they still need to calculate the scale values of mass and volume for the models of each of these bodies.

> **Note:** Calculating the scale mass of the planets and moons can be accomplished in various ways. The easiest method would be to simply tell your students the number of cotton balls that will represent the mass of each planet or moon. There are two ways you can give your students the mass of the planets or moons so that they can calculate the scale masses for themselves. Students can be given the planets' and moons' masses in terms of 10^{22} kg. Alternatively, if students are comfortable working with exponents, they can be given the planets' masses in kg. In either way, students can apply what they learned from Lesson 1 on calculating ratios. (If one cotton ball equals the mass of the Earth, this is the same relationship as the actual mass of the Earth; so 1 cotton ball represents 597 kg x 10^{22} or 5.97 kg x 10^{24}.) Students should then be able to conceptually explain how to then calculate the number of cotton balls that will represent the other planets. Again, this will reinforce student conceptual understanding, communication skills, and problem solving skills.

What is the actual mass of Earth? (597 kg x 10^{22} or 5.97 x 10^{24} kg)

We know that 1 cotton ball will represent the mass of Earth. This allows us to set up a ratio.

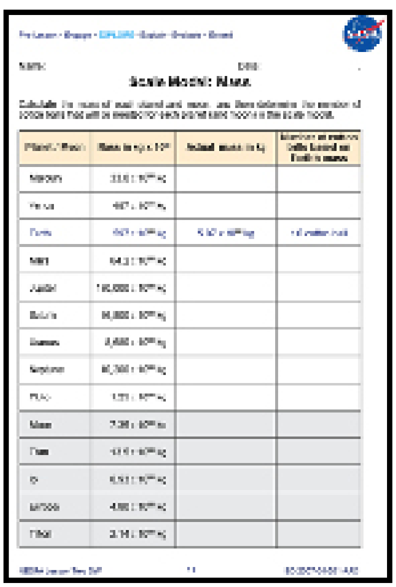
$$\frac{\text{Mass of the Earth in the scale model}}{\text{Actual mass of the Earth}} = \frac{1 \text{ cotton ball}}{597 \text{ kg} \times 10^{22}}$$

If one cotton ball represents 597 kg (x 10^{22}) and we know that the actual mass of Jupiter is 190,000 kg (x 10^{22}) or 1.90 kg x 10^{27}, *how can we find the number of cotton balls that will represent Jupiter?*

Ask the students how they can find that relationship. Allow the students to work together using calculators if necessary. After some groups have found a solution, have them share their result with the class. The following questions are examples of the types of questions that will help to strengthen students' math communication skills.

- How did you find your solution? Explain.
- How do you know it is correct? Why did you perform (a particular) calculation that way?
- Do other students have questions about how your group solved the problem? Does anyone disagree with your group's solution?
- Does your answer make sense? (For example, should the mass model of Jupiter use more or less cotton balls than the mass model of Earth?)
- Do you think your strategy would apply to other situations? How? (You can provide "what if scenarios" to help students generalize to other situations.)
- How do different students' strategies for solving the problem compare? Which strategy do you like the best? Why?

Some students may see that 190,000 divided by 597 will give the number of cotton balls that will represent the mass of Jupiter in the model. If they can make that connection, then they can use that relationship to find the number of cotton balls used to represent the mass of all of the planets in the model.


Scale Model: Mass

Another method is to use ratio and proportion, which was introduced in Lesson 1. Step-by-step examples of using ratio and proportion are included in the print materials section of this lesson for each of the two possible data sets: **Calculating Mass Scale <u>Without</u> Scientific Notation** (TG pp.25-27) or **Calculating Mass Scale <u>With</u> Scientific Notation** (TG pp.28-29).

Once students have found a method that gives them the correct result, they can apply that method to the rest of the planets to determine what number of cotton balls will be used to represent the mass of each planet or moon. Students will record their answers on the **Scale Model: Mass worksheet** (SW p.10).

Note: Most masses of the model planets will be less than one. Students will need to calculate the mass of the planets to the nearest <u>thousandth</u>.

Calculating Mass Scale Without Scientific Notation

Earth's mass is: 5,970,000,000,000,000,000,000,000 kg

Explain to the students that to simplify their calculations, they will work with 597 and they will have to remember that 22 zeros follow the 597.

You can demonstrate to the students that this will not interfere with their calculations as long as they follow the same rule for each planet because each planet's mass can be said to end in 22 zeros. In their calculations, the zeros will cancel out. To illustrate this, we use the following example.

Jupiter's mass is 190,000 followed by 22 zeros. To calculate the mass of Jupiter in terms of Earth's mass, we use the following equation.

$$\frac{\text{Jupiter scale model mass}}{\text{Actual Jupiter mass}} = \frac{\text{Earth scale model mass}}{\text{Actual Earth mass}}$$

We represent Earth's mass using 1 cotton ball.

We let "x" be the number of cotton balls that represent the mass of the scale model of Jupiter:

$$\frac{x}{1{,}900{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ kg}} = \frac{1 \text{ cotton ball}}{5{,}970{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ kg}}$$

Ask students how they would solve for x?

To solve for x, we cross multiply.

$$5{,}970{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ kg} \cdot x = 1{,}900{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ kg} \cdot 1 \text{ cotton ball}$$

Next, we divide both sides of the equation by 5,970,000,000,000,000,000,000,000 kg.

$$\frac{5{,}970{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ kg} \cdot x}{5{,}970{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ kg}} = \frac{1{,}900{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ kg} \cdot 1 \text{ cotton ball}}{5{,}970{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ kg}}$$

The 22 zeros and the kilograms (kg) cancel.

$$\frac{\cancel{5{,}970{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ kg}} \cdot x}{\cancel{5{,}970{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ kg}}} = \frac{1{,}900{,}00\cancel{0{,}000{,}000{,}000{,}000{,}000{,}000{,}000} \text{ kg} \cdot 1 \text{ cotton ball}}{5{,}970\cancel{{,}000{,}000{,}000{,}000{,}000{,}000{,}000} \text{ kg}}$$

$$x = \frac{190{,}000 \text{ cotton balls}}{597}$$

$$x = 318.2 \text{ cotton balls}$$

$$\mathbf{x \approx 318 \text{ cotton balls}}$$

So x (the model mass of Jupiter) is approximately equal to 318 cotton balls.

Explain to the students that they will get the same result if they omit the 22 trailing zeros for each planet's calculation since those zeros will cancel out. Students can be given the planetary masses in terms of 10^{22} and can ignore the 22 trailing zeros to calculate the number of cotton balls for each planet. (See the example on the following page.)

Example: If you give Earth's mass as 597 and Jupiter's mass as 190,000, then the result is the same as in the previous calculation.

$$\frac{\text{Jupiter scale model mass}}{\text{Actual Jupiter mass}} = \frac{\text{Earth scale model mass}}{\text{Actual Earth mass}}$$

$$\frac{x}{190{,}000 \text{ kg}} = \frac{1 \text{ cotton ball}}{597 \text{ kg}}$$

$$597 \text{ kg} \cdot x = 1 \text{ cotton ball} \cdot 190{,}000 \text{ kg}$$

$$\frac{\cancel{597 \text{ kg}} \cdot x}{\cancel{597 \text{ kg}}} = \frac{1 \text{ cotton ball} \cdot 190{,}000 \cancel{\text{kg}}}{597 \cancel{\text{kg}}}$$

$$x = \frac{190{,}000 \text{ cotton balls}}{597}$$

$$\mathbf{x \approx 318 \text{ cotton balls}}$$

The result is the same; we simply eliminated the step of canceling 22 zeros.

Calculating Mass Scale With Scientific Notation

If students are familiar with exponents, then they may be able to do the preceding calculations in terms of scientific notation.

Explain to students that scientists and mathematicians use shorthand for numbers that are very large or very small. They use scientific notation, which lets us write a number in an equivalent form without the trailing zeros.

For example, six billion is 6 multiplied by 1 billion.

$$6{,}000{,}000{,}000 = 6 \cdot 1{,}000{,}000{,}000$$

One billion is equivalent to 10 multiplied by itself 9 times (meaning there are 9 zeros). We can write this in *exponential* form: 10^x.

$$\begin{aligned} 1{,}000{,}000{,}000 &= 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \\ &= 10^9 \end{aligned}$$

We can rewrite the original number 6 billion using exponents.

$$\begin{aligned} 6{,}000{,}000{,}000 &= 6 \cdot 1{,}000{,}000{,}000 \\ &= 6 \times 10^9 \end{aligned}$$

Large numbers, such as planetary masses, are often written using scientific notation. Scientific notation is helpful because expressions containing exponents and the same base (10) can be easy to simplify.

Note: Most of the planets have a mass less than that of Earth. Students need to be careful when canceling zeros. Unless the students are comfortable using scientific notation and working with exponents, it may be best to calculate the masses as a class or simply to demonstrate this method to the students.

Students can use the scientific notation forms of planetary masses to determine the mass of the planets in the scale model.

$$\frac{\text{Jupiter scale model mass}}{\text{Actual Jupiter mass}} = \frac{\text{Earth scale model mass}}{\text{Actual Earth mass}}$$

In the scale model we represent Earth's mass using 1 cotton ball. We let "x" be the number of cotton balls that represent the scale mass of Jupiter.

$$\frac{x}{1.9 \times 10^{27}\ \text{kg}} = \frac{1\ \text{cotton ball}}{5.97 \times 10^{24}\ \text{kg}}$$

Students can solve for x by cross multiplying.

$$5.97 \times 10^{24}\ \text{kg} \cdot x = 1.9 \times 10^{27}\ \text{kg} \cdot 1\ \text{cotton ball}$$

To solve for x, divide both sides of the equation by 5.97×10^{24} kg.

$$\frac{5.97 \times 10^{24}\ \text{kg} \cdot x}{5.97 \times 10^{24}\ \text{kg}} = \frac{1.9 \times 10^{27}\ \text{kg} \cdot 1\ \text{cotton ball}}{5.97 \times 10^{24}\ \text{kg}}$$

The kilograms (kg) cancel. **Note:** $\frac{10^{27}}{10^{24}} = 10^{27-24} = 10^{3}$

$$\frac{\cancel{5.97 \times 10^{24}\ \text{kg}} \cdot x}{\cancel{5.97 \times 10^{24}\ \text{kg}}} = \frac{1.9 \times 10^{27-24}\ \cancel{\text{kg}} \cdot 1\ \text{cotton ball}}{5.97\ \cancel{\text{kg}}}$$

$$x = \frac{1.9 \times 10^{3} \cdot 1\ \text{cotton ball}}{5.97}$$

$$x = \frac{1{,}900 \cdot 1\ \text{cotton ball}}{5.97}$$

$$x = 318.2\ \text{cotton balls}$$

$$\mathbf{x \approx 318\ \textbf{cotton balls}}$$

So x (the scale model mass of Jupiter) is approximately equal to 318 cotton balls.

2.2 Calculating Circumference of a Great Circle for the Solar System Scale Model

Students will need to calculate the circumference of the planets in the scale model. In *What's the Difference*, the circumference of the planets is measured in kilometers. Ask the students:

What unit should be used to measure the circumference of the models? (centimeters)

Why would feet, yards, or meters be a poor choice? (too big)

To simplify the calculations and ensure that the model planets have a manageable volume, the circumference of the model of Pluto (the smallest model the students will be making) will be 1 cm.

If 1 cm will represent the circumference around a great circle on the model of Pluto and the actual circumference around a great circle on Pluto is 7,232 km, then ***what is the ratio*** *of the scale circumference of a great circle on Pluto to the actual circumference around a great circle on Pluto?*

Students should be able to generate the following ratio:

$$\frac{\text{Circumference of a great circle on the scale model of Pluto}}{\text{Actual circumference of a great circle on Pluto}} = \frac{1\text{ cm}}{7{,}232\text{ km}}$$

or

1cm : 7,232 km.

If 1 cm represents 7,232 km and the actual circumference around a great circle on Jupiter is 449,197 km, then how can we find the number of cm that will represent the circumference of a great circle on Jupiter?

Use the same process as before, allowing the students to work together using calculators if necessary and discussing student solutions and strategies.

Some students may see that 449,197 divided by 7,232 will give the number of cm that will represent the circumference around a great circle on Jupiter in the scale model. If they can make that connection, then they can use that relationship to find the scale volume of all of the planets in their model.

Another method is to use ratio and proportion, which was introduced in Lesson 1.

Using a ratio of 1 cm to the circumference of Pluto, the circumference of the rest of the planets can be calculated using ratio and proportion. In the following sample equation, "x" represents the circumference of the scale model of Jupiter—the number we want to find.

Sample Equation:

$$\frac{\text{Pluto scale model circumference}}{\text{Actual circumference of Pluto}} = \frac{1\text{ cm}}{7{,}232\text{ km}}$$

$$\frac{\text{Jupiter scale model circumference}}{\text{Actual circumference of Jupiter}} = \frac{x}{449{,}197\text{ km}}$$

$$\frac{1\text{ cm}}{7{,}232\text{ km}} = \frac{x}{449{,}197\text{ km}}$$

$$1\text{ cm} \cdot 449{,}197\text{ km} = x \cdot 7{,}232\text{ km}$$

$$\frac{1\text{ cm} \cdot 449{,}197\ \cancel{\text{km}}}{7{,}232\ \cancel{\text{km}}} = \frac{x \cdot \cancel{7{,}232\text{ km}}}{\cancel{7{,}232\text{ km}}}$$

$$\frac{449{,}197\text{ cm}}{7{,}232} = x$$

$$\mathbf{62.1\text{ cm}} \approx \mathbf{x}$$

Thus, the scale model circumference of Jupiter is approximately 62.1 cm.

By cross-multiplying and solving for x, students find that the circumference of the scale model of Jupiter is approximately 62.1 cm. Using ratio and proportion, the ratio for the circumference of the scale model of Pluto to the actual circumference of Pluto can be used to solve for the circumference of all of the planets in the scale model. Students will record their answers on the **Scale Model: Circumference** worksheet (SW p.11).

2.3 Building the Scale Model for Mass and for Circumference of a Great Circle

Note: Depending on the amount of materials you have available, the class can make one model of the solar system together or multiple models in groups. If several groups make models of the solar system, be sure to have them compare the models in terms of volume and density when they are done. This can help to cross check for accuracy.

Using the calculated masses and circumferences, the class (or each group of students) will need ~450 cotton balls, 9 plastic bags, and 9 twist ties to create models of the nine planets. Students will count out the number of whole cotton balls or divide cotton balls into pieces to represent the mass of the planets. (Using scissors to cut the cotton balls into pieces may be a more accurate and easier method for the students as opposed to tearing them apart.)

Next, the students will place the cotton into the plastic bags and begin to form a spherical planet model. They will need to make sure that each model has the correct circumference. Some of the planets will require a lot of squishing, while other planets will have more room in the bags. Students should use string and a ruler to measure the circumference of the model about multiple great circles in order to achieve as accurate a shape as possible.

Note: Because the models the students are making will be irregular and difficult to manage, it may be helpful to have the students practice measuring great circles using string or tape measures on basketballs or other solid spheres in the classroom. If possible, give them time to practice with different spheres and double-check to ensure that they are measuring the spheres at the widest points. One method would be to have several students measure the circumference of the same basketball and then have them compare their answers.

As some of the planets will be very small, emphasize the importance of teamwork. Students will probably need to work together in order to count, measure, and create the models. Students may also need to problem solve and use a method of trial and error to get the correct circumference of the planets. Encourage them to take their time and work together to achieve the best model they possibly can.

While in reality the planets are not perfect spheres, the models of the planets should appear as spherical as possible. Foil can be placed around the outside of the bag to help mold the shape, but it should not change the size of the planet.

As the students create the models, have them label the planets with masking tape or place them on labeled index cards. Index cards should also contain the planetary data collected from *What's the Difference*: diameter, circumference, mass, density, gravity, and composition.

Fraction Equivalents

The mass of each of the four planets (Mercury, Venus, Mars, and Pluto) is less than the mass of Earth. The students will calculate the scale model mass of these planets in terms of a decimal amount of a cotton ball. When the students use their calculations to determine how much of a cotton ball will represent the mass of a planet, it is best if they use fractions equivalent to the decimal amounts.

For example, the mass of the scale model of **Pluto** will be represented by **0.002** cotton balls. Essentially, this is **2 thousandths** of a cotton ball—in other words, a very little piece. This is a good time to talk about precision of measurement and what degree of precision is required for the model. If you ask students if it will be possible to have precisely 0.002 of a cotton ball, they should be able to see that this will not be possible in a practical sense and that they will have to estimate.

For **Mercury**, the scale model mass will be represented by 0.055 of a cotton ball. This is 55 thousandths or **approximately one-twentieth**. One-twentieth can be found by cutting a cotton ball into tenths and then cutting a tenth in half.

$$\frac{55}{1{,}000} \text{ is approximately equal to } \frac{50}{1{,}000}$$

$$\approx \frac{5}{100}$$

$$\approx \frac{1}{20}$$

For **Mars**, the mass of the scale model is 0.108 cotton balls (**approximately one-tenth**).

For **Venus**, the mass of the scale model is 0.816 (**approximately eight-tenths**).

3. COMPARING PLANETARY CHARACTERISTICS

When the scale models of the planets are complete, the students will need time to observe the differences between them. Spread the solar system model(s) and the information cards around the classroom and allow students to move around the room to each model. Students can record their observations on the **Scale Model: Observations worksheet** (SW p.12).

4. CALCULATING VOLUME AND DENSITY

In the ENGAGE portion of the lesson (TG pp.15-19), students were introduced to the formulas for calculating volume and density. These formulas will be implemented on the **Volume and Density** worksheet (SW p.13) as students first calculate the volume of each planet and then use volume and mass to calculate each planet's density.

SAMPLE: Calculating the volume and density of Earth

Step 1: Refer to the Lesson 2 Planet Data sheet to find Earth's diameter in km.

$$\text{Earth's diameter} = 12{,}755 \text{ km}$$

Step 2: Divide the diameter by 2 to find Earth's radius in km.

$$\text{Earth's radius} = 12{,}755 \text{ km} \div 2$$
$$= 6{,}377.5 \text{ km}$$

Step 3: Convert kilometers to meters using a unit ratio.

$$6{,}377.5 \cancel{\text{km}} \bullet \frac{1{,}000 \text{ m}}{1 \cancel{\text{km}}} = 6{,}377{,}500 \text{ m}$$

Step 4: Solve for volume.

$$V = \frac{4 \pi r^3}{3}$$
$$= \frac{4 \pi (6{,}377{,}500 \text{ m})^3}{3}$$
$$= \frac{4 \pi (2.59 \times 10^{20} \text{ m}^3)}{3}$$
$$\approx \frac{3.25 \times 10^{21} \text{ m}^3}{3}$$
$$\approx 1.08 \times 10^{21} \text{ m}^3$$

Step 5: Solve for density.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$
$$= \frac{597 \times 10^{22} \text{ kg}}{1.08 \times 10^{21} \text{ m}^3}$$
$$= \frac{5.97 \times 10^{24} \text{ kg}}{1.08 \times 10^{21} \text{ m}^3}$$
$$= \frac{5.5277 \times 10^{3} \text{ kg}}{\text{m}^3}$$
$$\approx 5{,}528 \text{ kg/m}^3$$

SW = student workbook TG = teacher guide EG = educator guide

Lesson 2 – EXPLAIN

• **Estimated Time:** 1 session, 50 minutes

• **Materials:**

- Students' notes from EXPLORE session (SW pp.8-10 and p.13)
- Density: Large Planets vs. Small Planets (SW p.14)
- Characteristics: Large Planets vs. Small Planets (SW p.15)
- Transparency #3: The Planets Ranked by Density (TG p.38)
- Graphing Resource—Student Guide (SW pp.16-19)
- Rulers
- Graph paper or adding machine tape
- Large chart or poster paper
- Calculators

---

1. CHARACTERISTICS OF LARGE AND SMALL PLANETS

Now that the students have created mass and volume scale models of the planets, they can make the connection to density and how it is related to the mass of planetary bodies.

To begin the discussion, ask the students:

What is wrong with the model in terms of composition?

What did they use for the composition of all of the scale model planets?

Are the real planets all made of the same material?

It is important that the students understand that the model is not accurate because in reality, the planets are not all composed of the same material.

Have the students review their notes from when they studied the scale model of the planets in the EXPLORE section (SW p.13). Allow the students to share what they observed and what questions they had. They will be using their observations on the planets to draw some conclusions later in this session. Ask the students:

What is the main difference in how the larger and smaller planets felt?

Students should respond that the larger planets felt looser or were not packed as tightly, but the smaller planets were very solid and tightly packed.

Density describes how tightly packed something is—i.e., how much matter is in a certain amount of space. You can use the analogy of students in a classroom. If you have 3 students in a classroom, the students could be spread out and the “density” of the classroom would be “low.” If you have 100 students in the same classroom, they would be packed together more tightly and the “density” would be “high.”

Have the students estimate the relative density of the planets by organizing them from greatest density to lowest density based on their model. Students can make a list individually or in groups and can use the **Density: Large Planets vs. Small Planets worksheet** (SW p.14).

Note: The values for the following discussion are shown on **Transparency #3: The Planets Ranked by Density** (TG p.38).

When the students have completed their list, write the planets on the board in order of most dense to least dense. Include the density next to each planet’s name. Next, add the masses of the planets to the board. Ask the students:

Do you notice a relationship between mass and density?

If a planet has a high mass, how does it rank in terms of density?

What if the planet has a lower mass?

To demonstrate this, circle the four planets with the highest mass. Ask the students:

How do these planets rank in terms of density? (More massive planets have lower densities. Less massive planets have higher densities.)

Is this what you would expect?

Misconception Alert!

The density and mass relationship of the planets can be counterintuitive. If density is a measure of matter per volume, it might make sense that higher mass would mean higher density. Part of this can be explained by the volume of the planets. The more massive planets are MUCH larger than the less massive planets. Therefore there is more space for their matter to spread out. This can also be explained when the students consider the composition of the planets.

Using the **Characteristics: Large Planets vs. Small Planets worksheet** (SW p.15), have students organize their observations by listing general characteristics about the two categories of planets. At the bottom of the worksheet, students should write conclusions they can make based on their observations.

The least dense planets are composed mostly of gas. The more dense planets are composed mostly of rock. Rock is much denser than gas. If the students do not recognize this, ask them to think about two balloons: one filled with air, the other filled with pebbles. Which one would have a higher density? (the balloon filled with pebbles)

The larger planets have more mass, which causes them to have more gravity. Because they have more gravity, they are able to hold onto lighter gasses that might escape from smaller planets or moons with less gravity. The higher gravity of the larger planets is the reason they are composed mostly of gas. (If the mass of a planet is too small, then a planet may not have enough gravity to hold on to heavy gases, like nitrogen and oxygen, that are needed for life.)

Ask the students to think of why it would be difficult for humans to visit the larger planets? Students may respond that the higher gravity would be a problem, which is true, but the main reason that humans would have difficulty visiting the larger planets is that the larger planets are composed of GAS. There is no surface for humans to land or work on, other than an incredibly dense and deep core.

However, the larger planets have several moons. Ask students to describe the characteristics of these moons based on the mass, density, composition, and gravity data they collected from *What's The Difference*. Students should see that the moons have the same properties of the smaller planets (less mass, less gravity, made of rock). The moons of the larger planets would allow humans to travel to the further regions of the solar system and still have a solid body on which to land and work.

Transparency #3: The Planets (and Moons) Ranked by Density

Planet (Moon)	Density kg/m^3	Mass $kg \times 10^{22}$	Composition	Gravity m/s^2
Earth	5,515	597	rock	9.80
Mercury	5,427	33	rock	3.70
Venus	5,243	487	rock	8.87
Mars	3,933	64.2	rock	3.71
Io	3,530	8.93	rock	1.80
Moon	3,350	7.35	rock	1.62
Europa	3,010	4.8	rock	1.31
Triton	2,050	2.14	rock	0.78
Titan	1,881	10.8	rock	1.35
Pluto	1,750	1.48	rock	0.58
Neptune	1,638	10,200	gaseous	11.15
Jupiter	1,326	190,000	gaseous	24.79
Uranus	1,270	8,680	gaseous	8.87
Saturn	687	56,800	gaseous	10.44

2. Determining Minimum and Maximum Mass Values for Human Exploration

Students are now ready to answer the scientific question and specify the values for the mass of planets and moons suitable for human visitation.

Ask the students to identify the four planets with the largest amount of mass. (Jupiter, Saturn, Uranus, and Neptune.)

Ask the students to explain what those four planets have in common that also sets them apart from the other planets. (They are gas planets.)

Have the students review their notes from the **Lesson 2 Planet Data Sheets** (SW pp.8-10).

How did they describe the surface fly-by?

What did the surface of these planets look like?

Did it look like there would be a good place for a crew vehicle to land on the surface?

Since it is not possible for a crew vehicle to land on the gas planets, humans will not be able to visit those planets. Students need to use this information to determine a minimum and a maximum mass of planets that are acceptable for humans to visit. This information will be helpful if other planets are discovered in the solar system or in other planetary systems around other stars. The mass of a new planet will help determine if it is possible for humans to land on its surface.

Using the masses of the planets, students will need to determine acceptable values of mass for human visitation. Although there is much debate regarding what precisely defines a planet, inform students that some scientists consider an object with less mass than Pluto orbiting the Sun to be an asteroid. *In fact, in August 2006, Pluto was reclassified as a dwarf planet, effectively reducing the number of "planets" in our solar system to eight.* Once planets become more massive, their gravity attracts lighter molecules and their composition becomes mostly gas. It is up to the students to decide minimum and maximum mass values, but they will have to justify their decisions.

Note: Results for the minimum and maximum masses will vary. Accept all answers as long as the gas giants do not fall into the category of acceptable values of mass and the rocky planets are not outside the acceptable values of mass. The students should justify their answers and back up their decisions with evidence.

Note: The minimum and maximum mass values determined by the students will include planets with less gravity. They will need to keep in mind that astronauts will need to adapt to this as well. Research has shown that microgravity can have negative effects on the human body. Humans who spend extended periods of time in microgravity experience a weakening of muscles because the muscles are not working against the gravity of Earth. Heart muscles weaken as well because it is easier to pump blood when there is no gravity. Because the muscles are not pushing and pulling on the bones in the body, bones get smaller and weaker over time. Scientists have found that exercise is one of the most important activities that astronauts must do in microgravity in order to stay healthy. If students are considering sending humans to planets with less gravity, then they must plan for how the astronauts can remain healthy at their destination. Currently the longest an astronaut has stayed in microgravity is 418 days—1 year and 53 days. Students will need to consider the effects of microgravity on humans over long periods of time if they decide to send humans to the outer planets.

Students can work in groups to complete this task. They can use large poster or chart paper to define their minimum and maximum values, justify their values, and graph which planets and moons have masses that fall within their acceptable values of mass and which planets do not. Students should use a number line and shade in values that lie between their minimum and maximum values. The mass of the planets and moons should be included on the number line to demonstrate which planets and moons fall within acceptable values, and which do not. Students should also include a graph to show how many planets fall within the acceptable values and how many do not. [Review the **Graphing Resource—Student Guide** (SW pp.16-19) with the students so that they can choose an appropriate graph. A bar graph or a pie chart would be appropriate.]

Groups will need to present their poster to the class. They should explain how they determined their acceptable values and how they decided to graph their data. Ask the following questions to ensure that each group communicates all of the important information.

- How did you choose your minimum and maximum mass values?
- What adaptations might need to be made for humans who land on a planet that has a mass value that lies between the minimum and maximum values?
- Which planets have masses that are between your minimum and maximum masses?
- Explain how you decided to use your particular data set and graph.
- How do you know your graph is accurate?
- Do other students have questions about how your group graphed the data? Does anyone disagree with your group's graph?
- Does your graph make sense? (For example, are there more planets with an acceptable mass than there are planets that do not have an acceptable mass?)
- How do different students' strategies for graphing the data compare? Which strategy do you like the best? Why?

3. COMPARING OTHER BODIES TO EARTH

By comparing the value of gravity and the density of other planets to those of Earth and placing those values on a number line, we not only see how they compare to Earth but by how much.

It is very common for scientists to generate data, such as mass or density, in terms of the Earth's mass or density. Students saw an example of how scientists calculate the distance to the Sun in terms of the Earth's distance to the Sun (Astronomical Units).

Have students calculate the density and gravity of the planets and moons in terms of the Earth's gravity and density in order to compare their values on number lines.

Students first need to round all values of the planet and moon gravities to the nearest whole number. Next, have students express the gravity of each body as a fraction of the Earth's gravity.

$$\frac{\text{Mercury's Gravity (kg/m}^2\text{)}}{\text{Earth's Gravity (kg/m}^2\text{)}} = \frac{4}{10}$$

The gravity on Mercury is four-tenths of the gravity of Earth.

In order that all fractions have the same denominator (a common denominator) and have an accurate value for the gravity on Pluto, students will need to convert these fractions to hundredths. They should understand that multiplying both the top and bottom by 10 gives an equivalent fraction because 10 divided by 10 is equivalent to 1.

$$\frac{\text{Mercury's Gravity (kg/m}^2\text{)}}{\text{Earth's Gravity (kg/m}^2\text{)}} = \frac{4}{10} \cdot \frac{10}{10}$$

$$= \frac{40}{100}$$

The gravity on Mercury is forty-hundredths of the gravity of Earth.

Have students calculate these values for all of the planets and moons. These values can then be placed on a fractional number line. Students should label the names of the planets and moons so that they can compare the relative values of the gravity on each body.

Note: Number lines can be created on graph paper, or for longer lines, adding machine tape is also helpful.

The fraction that represents the gravity on each planet or moon can be converted to a decimal by dividing the numerator by the denominator. For example, 40/100 is in fact 40 divided by 100.

$$40 \div 100 = 0.40$$

The gravity of Mercury is 0.40 (forty-hundredths) of the gravity of Earth. This decimal can also be represented as a percent. Percent means "divided by one hundred." Therefore, the decimal gives a percentage value, namely the percent of gravity on a planet or moon compared to Earth. For example, the percent of gravity on Mercury is 40% of the gravity on Earth.

The same process can be used for calculating the density of a planet. It is often useful to express the value of the density of a planet compared to Earth's density (Earth = 100%) in terms of a decimal or a percent. These percentages for all of the planets can then also be placed on a number line and labeled.

$$\frac{\text{Density of Mercury (kg/m}^3\text{)}}{\text{Density of Earth k(g/m}^3\text{)}} = \frac{5{,}427}{5{,}515} = 0.98 \quad (98\%)$$

The density of Mercury is 98% the density of Earth. Students will calculate these values for all of the planets and the moons and graph the data on a number line for percentages.

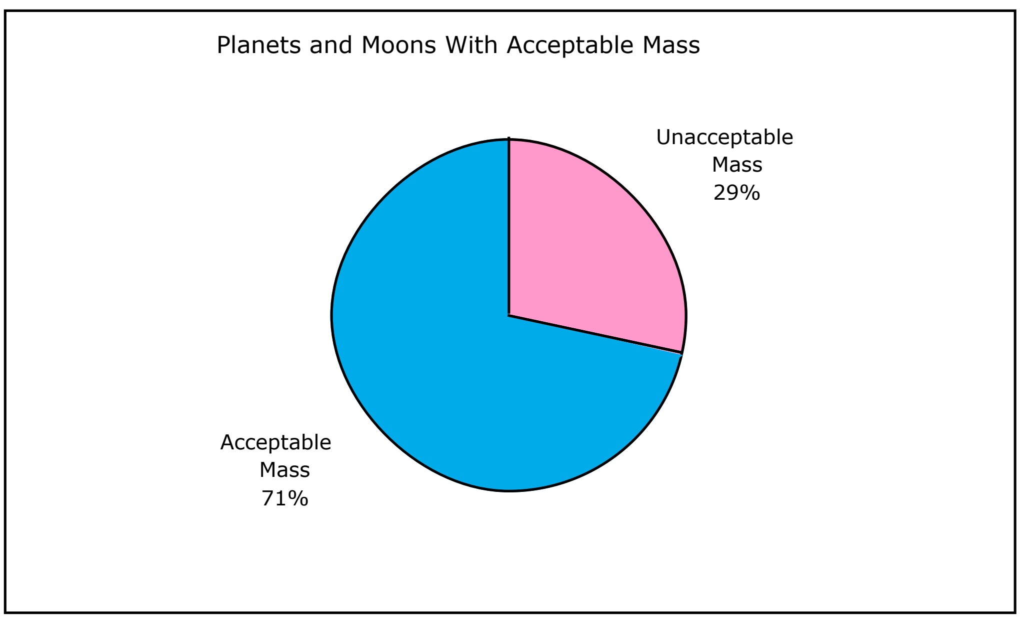
If the density and gravity number lines are compared, students should notice an interesting relationship. **Several planets with lower densities have much higher gravities than the other planets.**

Answer Key: EXPLAIN

Setting a Minimum and Maximum Mass

As long as the mass values are reasonable and justified, they are acceptable. The main goal of the activity is to demonstrate that the masses of the gas giants rule them out as potential destinations. The following are two possible graphs: a table and a pie chart.

Acceptable Mass	Unacceptable Mass
Mercury	Jupiter
Venus	Saturn
Earth	Uranus
Mars	Neptune
Pluto	
Moon	
Titan	
Io	
Europa	
Triton	

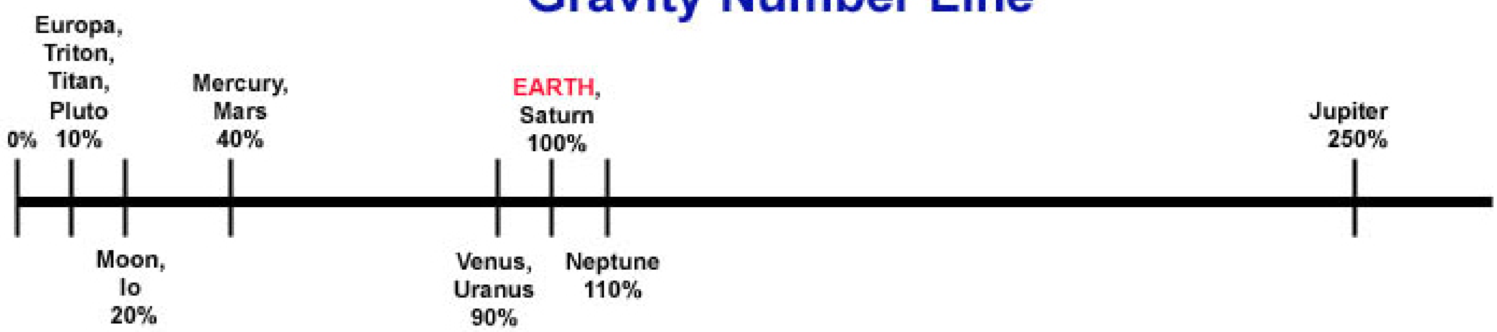


Answer Key: EXPLAIN

GRAVITY — COMPARING OTHER BODIES TO EARTH

Planet / Moon	Gravity m/s²	Rounded to Whole Number	Decimal of Earth's Gravity	Percent of Earth's Gravity
Jupiter	24.79	25	2.5	250%
Neptune	11.15	11	1.1	110%
Saturn	10.44	10	1.0	100%
Earth	9.80	10	1.0	100%
Uranus	8.87	9	0.9	90%
Venus	8.87	9	0.9	90%
Mars	3.71	4	0.4	40%
Mercury	3.70	4	0.4	40%
Io	1.80	2	0.2	20%
Moon	1.62	2	0.2	20%
Titan	1.35	1	0.1	10%
Europa	1.31	1	0.1	10%
Triton	0.78	1	0.1	10%
Pluto	0.58	1	0.1	10%

Gravity Number Line

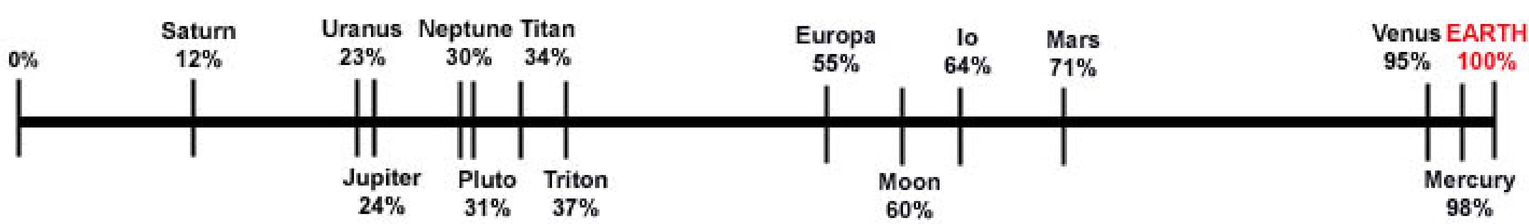


Answer Key: EXPLAIN

DENSITY — COMPARING OTHER BODIES TO EARTH

Planet (Moon)	Density kg/m^3	Decimal of Earth's Density	Percent of Earth's Density
Earth	5,515	1.00	100%
Mercury	5,427	0.99	98%
Venus	5,243	0.95	95%
Mars	3,933	0.71	71%
Io	3,530	0.65	64%
Moon	3,350	0.61	60%
Europa	3,010	0.54	55%
Triton	2,050	0.38	37%
Titan	1,881	0.34	34%
Pluto	1,750	0.36	31%
Neptune	1,638	0.29	30%
Jupiter	1,326	0.23	24%
Uranus	1,270	0.23	23%
Saturn	687	0.11	12%

Density Number Line



SW = student workbook TG = teacher guide EG = educator guide

Lesson 2 – EVALUATE

- **Estimated Time:** 1 session, 40 minutes
- **Materials:**
  - Student notes, observations, number lines, and range posters
  - Graphing Rubric (TG p.48)
  - Problem Solving Rubric (TG p.49)

To reflect on and review the lesson, lead the class in the following discussion.

Check for understanding:

1. What is the difference between weight and mass?
2. What do you need to have in order to have weight?
3. What is a number interval? Give an example of a situation where you would use a number interval.
4. What is a great circle?
5. What is circumference?
6. What is diameter?
7. What is density?
8. Describe the properties of the larger planets.
9. Describe the properties of the smaller planets.
10. Name something that astronauts must do to counteract the negative effects of experiencing microgravity.

Reflection:

1. What was challenging about making a scale model of planet mass and volume?
2. What was different about the planet size/distance scale models?
3. What did the class determine was an appropriate number interval of density for planets that humans may visit? Why?

4. What did the class determine was an appropriate number interval of mass for planets that humans may visit? Why?

5. Which planets, by virtue of their density and mass, were ruled out for human visits? Why?

6. What are some challenges that astronauts may face when they land on planets with different surface features or compositions than that of Earth?

7. What was some new or interesting information you learned in this lesson?

8. What new questions do you have about our solar system and the planets and moons?

> **Note:** A **Graphing Rubric** and **Problem Solving Rubric** are included in this section to help evaluate students' work in this lesson. (TG pp.48-49)

Preparing to move on:

1. Based on the density of the planets and your number interval of values for density, where can we not send humans in our solar system? Why?

2. What other challenges might rule out what planets humans may visit?

3. There are several moons of larger planets that are possible places for humans to visit. What advantages might exist for visiting a moon instead of a planet? What would be some disadvantages?

Brief closing assignment:

The following can be given as a brief, one paragraph writing assignment. Students can respond on index cards (which keep responses concise) or in a journal. Alternatively, students can discuss their answers in pairs or small groups and report their answers back to the class.

- What did you learn during this lesson?
- Based on your experience in this lesson, where do you think we should send humans in our solar system?
- What else do you need to know in order to make a recommendation?
- How will you gather that information?

In the next lesson, students will calculate travel times to the planets and moons in the solar system to determine mission lengths for each possible destination.

Graphing Rubric

Student graphs can be assessed with the following rubric.

4	• All data is graphed extremely accurately. Decimals and fractions are taken into account. • Graph is titled and all axes are correctly and neatly labeled. • Graph includes a consistent scale on the y-axis. • Graph type is appropriate for data used. • Choices for graph type, scale, and units are fully justified and related to the data.
3	• All data is graphed accurately. Decimals and fractions were rounded to whole numbers. • Graph is titled and all axes are labeled. • Graph includes a consistent scale on the y-axis. • Graph type is appropriate for data used. • Choices for graph type, scale, and units are justified and may be related to the data.
2	• Data is graphed somewhat accurately. Decimals and fractions were ignored. • Graph is missing either title or axis labels. • Graph includes a consistent scale on the y-axis. • Graph type is somewhat appropriate for data used. • Choices for graph type, scale, and units are not justified and/or may not be related to the data.
1	• Data is not graphed accurately. • Graph does not have a title or axis labels. • Graph does not have a consistent scale for y-axis. • Graph type is inappropriate for data used. • Choices for graph type, scale, and units are not justified and are not related to the data.

Problem Solving Rubric

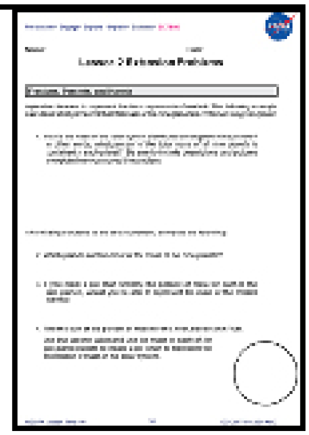
Problem-solving assignments and presentations can be assessed with the following rubric.

4	• Answers were calculated correctly to an appropriate degree of accuracy (rounded to a decimal place or whole numbers where specified). • Answers are fully explained and justified in detail. • All steps of the problem are explained in detail. • Information supplied by the students is accurate and the source of the information is given. • Picture that accompanies problem is relevant, labeled, and demonstrates how the problem was solved. • Written explanation completely outlines the problem and the solution.
3	• Answers were calculated correctly, but to an inappropriate degree of detail (rounded to whole numbers or not rounded where it was appropriate). • Answers are explained and justified. • All steps of the problem are explained. • Information supplied by the students is accurate, but the source of the information is not given in detail. • Picture that accompanies problem is somewhat relevant, may or may not be labeled, and somewhat demonstrates how the problem was solved. • Written explanation outlines the problem and the solution.
2	• Answers were mostly calculated correctly. • Answers are stated clearly but not explained or justified • All steps of the problem are not fully explained. • Information supplied by the students may not be accurate and the source of the information is not given. • Picture that accompanies problem is not relevant, is not labeled, or does not demonstrate how the problem was solved. • Written explanation does not clearly outline the problem and the solution.
1	• Answers were not calculated correctly. • Answers are not stated clearly and are not explained or justified. • Steps of the problem are not explained. • Information supplied by the students is not accurate. • No picture. • Written explanation does not outline the problem or the solution.

SW = student workbook TG = teacher guide EG = educator guide

Lesson 2 – EXTEND & APPLY (optional portion of lesson)

- **Estimated Time:** 1 session, 40 minutes
- **Materials:**
  - Lesson 2 Extension Problems (SW pp.20-23)
  - Problem Solving Teacher Resource (TG pp.52-54)
  - Helping Students Communicate Math Teacher Resource (EG pp.6-7)
  - Paper for student work
  - Rulers, tape measures, yardsticks or metersticks
  - (Optional) Calculators



Have students work on the **Lesson 2 Extension Problems** (SW pp.20-23). These problems are multi-step open-ended challenges. Some will require the students to measure lengths inside the classroom and then apply what they know about scale and ratio and proportions. Students may choose the units they work with, as long as they are appropriate. The problems can be done individually, in groups, or as a class.

You may want students to accompany each solution with a written and graphical explanation of how the problem was solved. Review the **Problem Solving—Teacher's Resource** and the example (TG pp.52-54) with your students before having them complete their own write up.

Fractions, Percents, and Planets

A pie chart is a way to represent fractions or percents of a whole. The following example calculates what percent of the total mass of the nine planets is contained in a given planet.

Ask students how they think the mass of the solar system is distributed amongst the nine planets. *What percent of the total mass of the planets is contained in Jupiter? What percent is in Earth? What about the other planets?*

Have students work in pairs or groups to come up with a strategy for solving this problem. Use the **Problem Solving—Teacher's Resource** and the **Helping Students Communicate Math—Teacher's Resource** (EG pp.6-7) to guide students in solving this problem and communicating their solutions.

There are a number of ways to solve this problem. The following is one possible solution.

- Using the planetary data charts, add the masses of all nine planets in our solar system.
- Calculate what fraction of the total mass of the planets is contained in each planet.

> For example, the fraction for Mercury is given by:
>
> $$\frac{\text{Mass of Mercury}}{\text{Total Mass of Planets}} = \text{Fraction of total mass contained in Mercury}$$

- Convert each fraction to a decimal.
- Convert the decimal to the percent of total mass of the planets that is contained in each planet.

After solving this problem, have students complete the follow-up questions and activities, which include the creation of pie charts.

Note: Introductions to pie graphs are included in the **Graphing Resource—Student Guide** (SW pp.16-19). This resource includes instructions on how to make accurate pie charts.

Think About It / Write About It / Discuss It Questions

1. What interval of values for the mass has surface conditions suitable for humans to visit? How did you come to this conclusion?
2. How do the surface conditions of low mass planets and moons compare to the surface conditions of high mass planets?
3. For the planets/moons that fall outside of the mass values habitable for humans, what are the factors that make these planets/moons difficult for humans to visit?
4. Choose one planet or moon inside the habitable interval of values. What challenges will the surface conditions of this body present to human visitation?
5. If humans were to visit a planet that was not composed of rock, what adaptations would they have to make?
6. How have humans adapted to study areas on Earth that are not composed of rock?

Problem Solving

Teacher's Resource

During the course of this unit, students will be presented with multi-step, open-ended challenges. The problems can be solved in a variety of ways, and there will often be multiple solutions. The problems can be done individually, in groups, or as a class.

Each problem can be accompanied by a written explanation and a picture explaining how the problem was solved. Students can use the following outline to explain their work in written form:

1. Restate the problem. What are you trying to find out?
2. What information do you have? What information do you need to find your answer? Explain how you got the information and record it.
3. Estimate what you think the answer will be. How do you know your estimate is reasonable?
4. Show your work. Include all calculations you made in order to solve the problem—even the ones that did not work.
5. Explain HOW you solved the problem. Step-by-step, what did you do? Use transitions like first, next, then, and finally.
6. State your answer. Explain HOW you know it is correct. Does it make sense? Why?
7. Draw a picture to go along with the problem. Label sizes and distances.

When you finish, read over your work. Pretend you are explaining this problem to someone younger than you.

- Is it clear?
- Does it make sense?
- Did you explain the problem and the answer well?

Example: Scale Movie Stars

Some fantasy characters, such as Hobbits from Lord of the Rings, or Hagrid from the Harry Potter series are on different scales than humans. The following calculations will demonstrate how an everyday object would need to be changed to fit the scale size of a character.

Hobbits are known as Halflings. They are about half the size of a human. Hagrid, however, is half-giant because he had a Giantess Mother. He is about twice the size of a human.

If your teacher became a Hobbit, estimate how tall he or she would be. Estimate how tall your teacher would be if he or she were Hagrid's size. Measure your teacher and calculate his or her Hobbit and Hagrid heights. If possible, mark the Hobbit height, Hagrid height, and actual height of your teacher on the wall or chart paper.

Sample Write Up:

1. I am going to calculate the height my teacher would be if she was a Hobbit or if she was a half-giant like Hagrid.

2. I know that Hobbits are half the size of humans, and I know that Hagrid is twice the size of a human. In order to solve the problem, I need to know my teacher's height. I will use a meter stick and measure her. My teacher is 1.75 meters tall.

3. I estimate that as a Hobbit my teacher will be less than a meter tall because Hobbits are much smaller. I think that as Hagrid my teacher will be over 3 meters tall because Hagrid is much bigger.

4. Hobbit Height:

   1.75 meters • 1/2 = teacher's Hobbit height

   1.75 meters • 0.5 = 0.875 m

   My teacher's Hobbit height = 0.875 m

   Hagrid Height:

   1.75 m • 2 = teacher's Hagrid height

   1.75 m • 2 = 3.5

   My teacher's Hagrid height = 3.5 m

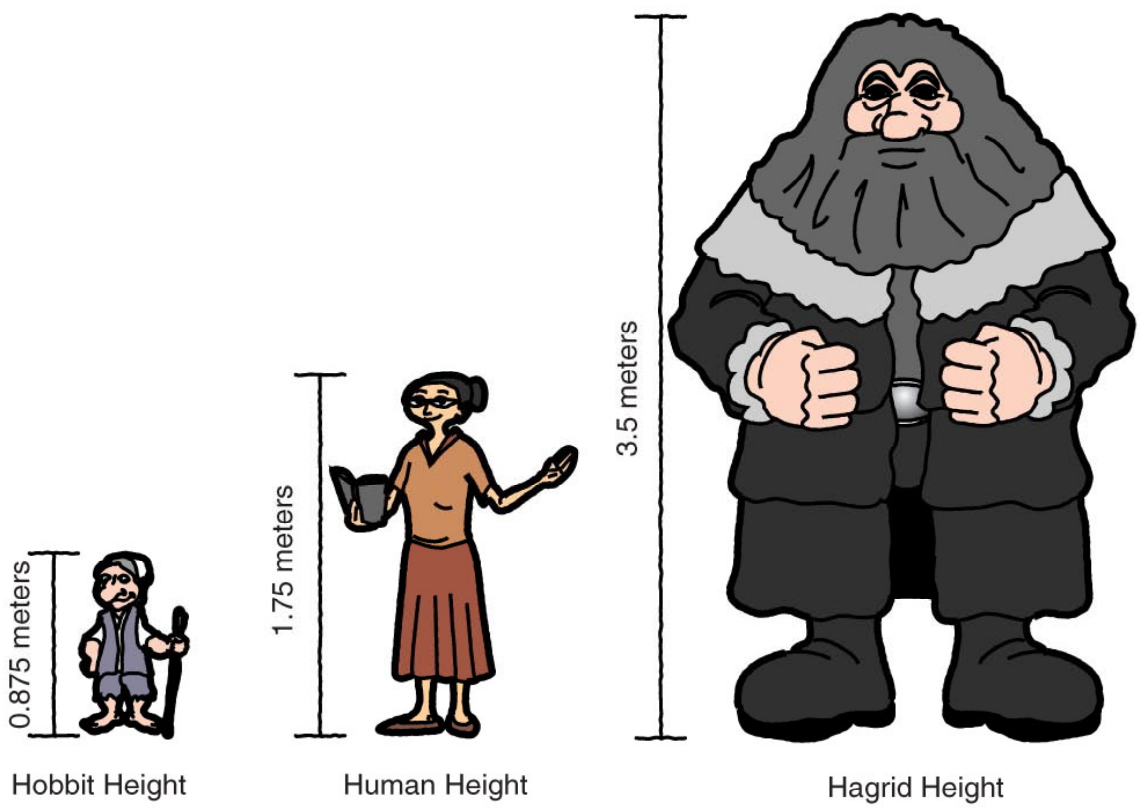
5. I solved the first part of the problem by multiplying my teacher's height by one-half. I solved the second part of the problem by multiplying my teacher's height by two.

   First, I solved for her Hobbit height. Hobbits are half the size of humans, so to get my teacher's Hobbit height I multiplied her normal height by one-half. I decided it would be easier to multiply decimals, so I multiplied 1.75 meters by 0.5 because 1/2 is equal to 0.5.

   Next, to get my teacher's Hagrid height, I multiplied her normal height by 2 because Hagrid is twice the size of a human.

6. I found that if my teacher were a Hobbit, she would be 0.875 meters tall because this is one-half of her normal height. I also found that if my teacher were like Hagrid, she would be 3.5 meters tall because this is two times her normal height. This makes sense because as a Hobbit she would be much smaller than her normal size, and as Hagrid she would be much bigger than her normal size. My estimates were pretty close. I was not off by that much.

7.



0.875 meters 1.75 meters 3.5 meters