



# **EXPLORING AERONAUTICS**

## **Part II**

### **Section 4**

# **Mathematics**



## Correlation to National Mathematics Standards

### Standard 1 Mathematics as Problem Solving

- develop and apply a variety of strategies to solve problems, with emphasis on multi-step and non-routine problems;
- verify and interpret results with respect to the original problem situation;
- generalize solutions and strategies to new problem situations;
- acquire confidence in using mathematics meaningfully.

### Standard 2 Mathematics as Communication

- model situations using oral, written, concrete, pictorial, graphical, and algebraic methods;
- reflect on and clarify thinking about mathematical ideas and situations.

### Standard 3 Mathematics as Reasoning

- recognize and apply deductive and inductive reasoning;
- understand and apply reasoning processes, with special attention to spatial reasoning.

### Standard 4 Mathematical Connections

- explore problems and describe results using graphical, numerical, physical, algebraic and verbal mathematical models or representations;
- value the role of mathematics in our culture and society.

### Standard 5 Number and Number Relationships

- understand, represent, and use numbers in a variety of equivalent forms in real world and mathematical problem situations;
- develop number sense for whole numbers, fractions, decimals, integers and rational numbers;
- represent numerical relationships in one and two dimensional graphs.

### Standard 6 Number Systems and Number Theory

- understand and appreciate the need for numbers beyond whole numbers;
- develop and apply number theory concepts in real world mathematical problem situations.

**Standard 7 Computation and Estimation**

- compute with whole numbers, fractions, decimals, integers and rational numbers;
- develop, analyze and explain procedures for computation and techniques for estimating;
- select and use appropriate method for computing from among mental arithmetic, paper-and-pencil, calculator and computer methods;
- use estimation to check the reasonableness of results.

**Standard 8 Patterns and Functions**

- describe and represent relationships with tables, graphs and rules.

**Standard 9 Algebra**

- understand the concepts of variable, expression and equation;
- represent situations and number patterns with tables, graphs, verbal rules and equations, and explore the interrelationships of these representations;
- analyze tables and graphs to identify properties and relationships.

**Standard 10 Statistics**

- systematically collect, organize and describe data;
- construct, read, and interpret tables, charts and graphs;
- develop an appreciation for statistical methods as powerful means for decision making.

**Standard 12 Geometry**

- identify, describe, compare and classify geometric figures;
- represent and solve problems using geometric models;
- develop an appreciation of geometry as a means of describing the physical world.

**Standard 13 Measurement**

- extend understanding of the process of measurement;
- estimate, make and use measurements to describe and compare phenomena;
- select appropriate units and tools to measure the degree of accuracy required in a particular situation;
- understand the structure and use of systems of measurement;
- extend understanding of the concepts of perimeter, area, volume, angle measure, capacity, and weight and mass.



## Goals and Objectives

### Goal 1

To familiarize students with the relationship between mathematics and aeronautics.

#### **Objectives**

The Learner will be able to:

- identify general ways in which mathematics is used in aeronautics;
- identify general ways in which mathematics is used to solve aeronautical problems.

### Goal 2

To use mathematics and mathematical tools to solve aeronautical problems.

#### **Objectives**

The Learner will be able to:

- use a calculator to convert Mach speeds to feet and miles per hour;
- use a calculator and measuring devices to create an aeronautical timeline;
- use the computer to check a hypothesis regarding the lift and drag of a wing;
- use a graph to compute the net force;
- use a measuring device, computer, calculator and graph to compute averages gathered on data from lift and drag tests as well as distance in flight tests.

### Goal 3

To introduce students to mathematical formulae used in aeronautics.

#### **Objectives**

The Learner will be able to:

- use volume and weight formulae for computing how much cargo can be carried;
- use the formula for averages to convert data;
- use the formula for aspect ratio to compute induced drag;
- use the Pythagorean Theorem for graphing and computing the four forces for flight;
- understand how the lift to drag ratio is used.



## Daily Lesson Planner

### Day 1 - 25

- It is recommended that you offer an instructional unit on measurement prior to using these mathematics lessons.
- Listed below are the mathematics lessons included in this section. The lessons are sorted according to the section on the CD-ROM **Exploring Aeronautics**, with which they are compatible. A brief idea of the content of each lesson is given as well.

CD-ROM Section	Lesson	Description
The Resource Center: History	Timeline Mechanics	design a timeline with appropriate units and tic marks
	Don't Let It Weigh You Down!	use a formula to determine how much cargo a lighter-than-air balloon can carry
The Hangar	Mach and Mile Mathematics with the X-15	convert speed to Mach numbers (and vice versa), convert feet to miles, learn about the speed of sound
Fundamentals of Aeronautics	The Aspect Ratio of Wings	use division to help determine how much lift and drag a wing will create
	Computing the Net Force	use addition and subtraction to calculate the net force, given two forces
	Graph the Four Forces	use a graph to understand the net force, given four forces
	Flying with Pythagoras	use the Pythagorean Theorem to find distance
	Wind Tunnel Averages	use averages to determine forces on a wind tunnel model
	Graphing Results	use a bar graph to analyze wind tunnel data



# Mathematics Lessons

## Timeline Mechanics

### Teacher-Led Exercise

**Connections:** This lesson involves using mathematics to create a timeline. It coordinates very well with Part II, Section 3: Social Studies. In Section 3: Social Studies, you will find lessons involving how to create and use a timeline based on events in aeronautical history. In Section 3, points on the timeline are chosen by a simple relationship between the years (for example, 1783 happened long before 1910). In this lesson, a method of calculating the length of and appropriate marks for a timeline is presented.

**Directions:** When designing your own timeline, you need to plan and measure carefully. It is always best to do a rough draft first before doing a final draft, so your final draft is neat and accurate. Use the following dates, and follow the steps below for making your own timeline.

Montgolfier	1783
Post	1933
Bleriot	1909
Yeager	1947
NASA	1958

**Step 1:** Find the earliest year from the dates you will be working with and round it down to the nearest decade (tens). Write the result below. This result will be the first date on your timeline.

**Step 2:** Find the latest year from the dates you will be working with and round it up to the nearest decade (tens). Write the result below. This result will be the last date on your timeline.

**Step 3:** Subtract the result from Step 1 from the result in Step 2. Write the result below. This result is the number of years on your timeline.



**Step 4:** Timelines are divided into units of time. Decide how you will show the years on your timeline.

**A)** Will you mark every year on your timeline? There may be too many years to show each one!

or

**B)** Will you mark every five years?

or

**C)** Will you mark every ten years?

or

**D)** Will you mark every twenty years?

or

**E)** Make another choice: \_\_\_\_\_

**Step 5:** Write your final choice here:

I will mark every \_\_\_\_\_ years on my timeline.

**Step 6:** Divide the total number of years on your timeline (your result from Step 3) by the number you have chosen in Step 5. The result will be how many marks you will draw on your timeline.

$$\frac{\text{result from Step 3}}{\text{result from Step 5}} = \text{number of marks you will have on your timeline}$$



**Step 7:** Decide how you will measure your timeline. Will your result from Step 5 be measured in:

A) inches                      B) half inches                      C) centimeters

or

Write in another possibility: \_\_\_\_\_

**Step 8:** Write your final choice here:

\_\_\_\_\_ years will equal one \_\_\_\_\_ on my timeline.  
result from Step 5                                      result from Step 7

**Step 9:** Calculate how long your timeline will be. Take your result from Step 6 and multiply it by your choice of measurement (your result from Step 7)

result from Step 6 X result from Step 7 = total length of your timeline

**Step 10:** On a piece of scratch paper, draw a rough draft of your timeline using the results from the steps above.



## Timeline Mechanics

### Student Exercise

**Directions:** When designing your own timeline, you need to plan and measure carefully. It is always best to do a rough draft first before doing a final draft, so your final draft is neat and accurate. Using the following dates, and follow the steps below for making your own timeline.

Sikorsky	1939
Zeppelin	1900
Quimby	1912
Coleman	1921
Sputnik	1957

**Step 1:** Find the earliest year from the dates you will be working with and round it down to the nearest decade (tens). Write the result below. This result will be the first date on your timeline.

**Step 2:** Find the latest year from the dates you will be working with and round it up to the nearest decade (tens). Write the result below. This result will be the last date on your timeline.

**Step 3:** Subtract the result from Step 1 from the result in Step 2. Write the result below. This result is the number of years on your timeline.



**Step 4:** Timelines are divided into units of time. Decide how you will show the years on your timeline.

**A)** Will you mark every year on your timeline? There may be too many years to show each one!

or

**B)** Will you mark every five years?

or

**C)** Will you mark every ten years?

or

**D)** Will you mark every twenty years?

or

**E)** Make another choice: \_\_\_\_\_

**Step 5:** Write your final choice here:

I will mark every \_\_\_\_\_ years on my timeline.

**Step 6:** Divide the total number of years on your timeline (your result from Step 3) by the number you have chosen in Step 5. The result will be how many marks you will draw on your timeline.

$$\frac{\text{result from Step 3}}{\text{result from Step 5}} = \text{number of marks you will have on your timeline}$$





## Timeline Mechanics

### Teacher-Led Exercise Key

- Step 1: 1783 -> 1780
- Step 2: 1958 -> 1960
- Step 3:  $1960 - 1780 = 180$  years
- Step 4: *answers will vary*
- Step 5: *answers will vary, example: 10*
- Step 6:  $180$  years /  $10$  years =  $18$  marks on your timeline
- Step 7: *answers will vary, example: inches*
- Step 8:  $10$  years will equal  $1$  inch on my timeline
- Step 9:  $18$  marks x  $1$  inch =  $18$  inches
- Step 10: *answers will vary*

### Student Exercise Key

- Step 1: 1900 -> 1900
- Step 2: 1957 -> 1960
- Step 3:  $1960 - 1900 = 60$  years
- Step 4: *answers will vary*
- Step 5: *answers will vary, example: 10*
- Step 6:  $60$  years /  $10$  years =  $6$  marks on your timeline
- Step 7: *answers will vary, example: inches*
- Step 8:  $10$  years will equal  $1$  inch on my timeline
- Step 9:  $6$  marks x  $1$  inch =  $6$  inches
- Step 10: *answers will vary*



## Mach and Mile Mathematics with the X-15



**Connections:** Portions of this lesson involve calculating the speed of an aircraft in terms of the Mach number. Mach numbers are used to define the “regimes” of flight: subsonic, supersonic, transonic and hypersonic. Another volume in the NASA series **Exploring Aeronautics**, entitled [The Regimes of Flight](#), explores the regimes of flight in depth and would provide an excellent follow-on unit for this section. [The Regimes of Flight](#) is available from the NASA Educator Resource Centers.

**Background:** The National Aeronautics and Space Administration (NASA) conducts space flight research to collect data on high speed aerodynamics. The X-15 aircraft was used extensively from 1959 - 1968 to fly faster and higher than any aircraft had before. The X-15 was the first aircraft to fly to the edge of space and return to Earth. The results of many X-15 test flights would later be used to design the Space Shuttle.

People who fly and work with high speed aircraft often use the term “Mach number” to describe the speed of an aircraft. “Mach number” was named after an Austrian physicist named Ernst Mach (1838-1916) who studied sound. A Mach number is special because it takes into account both the speed of the aircraft and the environmental condition of the air through which the aircraft is flying. The Mach number is



calculated by dividing the speed of the aircraft by the speed of sound at the altitude the aircraft is flying. Remember to keep the units for speed of the aircraft and the speed of sound the same!

$$\frac{\text{speed of the aircraft in miles per hour}}{\text{speed of sound in miles per hour}} = \text{Mach number}$$

If an aircraft is flying at Mach 1, we say that it is flying at the speed of sound. If an aircraft is flying at Mach 2, we say that it is flying twice the speed of sound. If an aircraft is flying at Mach 6, we say that it is flying six times the speed of sound.

Also, for these exercises you will need to remember that:

$$\text{one mile} = 5,280 \text{ feet}$$



# Mach and Mile Mathematics with the X-15

## Exercise 1

**Directions:** The X-15 had a very unique way of starting its flights. It was mounted on the belly of a B-52 and flown to an altitude of 45,000 feet, where it was launched at a speed of 500 miles per hour. A rocket in the X-15 would then provide thrust for roughly 120 more seconds, and then the X-15 would glide over 200 miles back to a runway. Navy Test Pilot, A. Scott Crossfield, was the first to fly the X-15.

Many test pilots flew the X-15 during the years it was tested, but two pilots broke world records during their flights. On August 22, 1963, NASA test pilot, Joseph A. Walker, flew the X-15 to an unofficial world altitude record of 354,200 feet. On October 3, 1967, Major William Knight, an Air Force Test Pilot, set the world speed record for winged aircraft. He flew 4,520 miles per hour. One year later, after 199 test flights, the X-15 was retired on October 24, 1968.

**Question 1:** The world altitude record, set by Test Pilot Walker, was 354,200 feet. What was his altitude in miles?

**Question 2:** The flight plan for Test Pilot Walker's record-breaking flight called for him to point the nose almost straight up and provide maximum rocket thrust after he was launched from the B-52. If the B-52 launched the X-15 at an altitude of 45,000 feet, how many feet up did he fly to break the altitude record?

**Question 3:** How many miles upward did he fly to break the altitude record?

He gained almost 90% (that is, almost all!) of his total altitude after he was launched, almost straight up, from the B-52 - in only 120 seconds! How would you like to have gone along for that ride?



**Question 4:** Can you think of a town that is between 58 and 67 miles away from your hometown? You can check the distance on a map. Does that town seem far away? Imagine going that distance, but going straight up! It might be fun to draw a picture of Test Pilot Walker's record-breaking flight!

**Question 5:** Test Pilot Major Knight had an equally exciting flight when he broke the world speed record! The old speed record was 4,486 miles per hour. By how many miles per hour did Major Knight beat the old record?

**Question 6:** The speed limit on most United States highways is 65 miles per hour. How many times faster did Major Knight fly than your car can legally go on the highway?

**Question 7:** The "speed of sound" is a measure of how fast sounds travel through the air. The "speed of sound" on earth, when the air temperature is 59 degrees, is 762 miles per hour. So, when a friend calls to you from across the schoolyard, the sound comes out of his/her mouth and enters your ears at 762 miles per hour!

The speed of sound changes as altitude and air temperature change. The speed of sound at the altitude at which Major Knight made his record-breaking flight was 87 miles per hour slower than on the ground. What was the speed of sound at Major Knight's altitude?

**Question 8:** Now that you know the speed of sound at Major Knight's altitude can you calculate the Mach number of the record-breaking flight?

**Question 9:** If Major Knight was flying Mach 6.7, how many times faster than the speed of sound was he flying?



# Mach and Mile Mathematics with the X-15

## Exercise 2

**Directions:** Based on what you learned in Exercise 1, Mach 1 would be one times the speed of sound. At sea level this is roughly 762 miles per hour.

Have you ever traveled at Mach 1?

Probably not! The fastest commercial jet airplanes in the United States generally fly below 500 miles per hour. However, there is a European airliner built by the French, the *Concorde*, that does fly just above Mach 1. If you have flown the *Concorde*, say from Paris to New York, then you are one of a fairly small group of people that has flown faster than the speed of sound!

Most of us have had to be content with driving, riding or flying at less than Mach 1. To see just how far below Mach 1 we generally travel, answer the following questions.

Remember that the speed of sound changes for two reasons. It changes according to altitude and the environmental condition of the air. You will need the following table for your calculations:

Altitude Range	Air Environmental Conditions	Speed of Sound
sea level	59 degrees F	762 miles per hour
20,000 - 30,000 feet	-30 degrees F	693 miles per hour
top of the atmosphere	-67 degrees F	662 miles per hour
350,000 - 360,000 feet		675 miles per hour
outer space	there is no air!	0

**Question 1:** In the United States the maximum speed limit on a freeway is 65 miles per hour. Assume that this freeway runs right along the ocean and it is a cool day. At what Mach number are you driving if you are driving at the speed limit?



**Question 2:** Say that you are a very accomplished mountain climber and you have decided to climb Mount Everest, the highest mountain on earth. It takes you many days, but finally you are standing at “the top of the world”! While you are standing there, trying to keep warm in -30 degree F weather, you see an F-14 flying at the same altitude you are standing! As she whizzes past you, the pilot gives you a “thumbs up” to congratulate you for making it to the top. If the pilot was flying at Mach 1, use the table above to determine how many miles per hour he/she was flying. Hint: Mt. Everest is 29,028 feet tall.

**Question 3:** How many miles per hour slower did the pilot of the F-14 have to go to achieve Mach 1 at the altitude of Mt. Everest, than you would have to go at sea level?

**Question 4:** In Exercise 1, Test Pilot Joseph Walker, broke a speed record in the X-15 at an altitude of 354,200 feet. If you were still standing on Mr. Everest, how many feet higher would Test Pilot Crossfield have been than you?

How many miles would that be?

If you were to travel the same number of miles by land from your hometown, where would you be? (Use a map to help you determine your answer.)



**Question 5:** The Space Shuttle flies at approximately 3,111 miles per hour right before it escapes from our atmosphere and enters outer space. What Mach number is this?

A tricky question: If the Space Shuttle flies 17,000 miles per hour in space, what is its Mach number? (Hint: Remember that the Mach number is a representation of the speed of sound. Think carefully about what affects the speed of sound!)

**Question 6:** A world class marathon runner can easily run at 15 miles per hour. What mach number is this? Assume he/she is running on the beach.

If he/she were running on the top of Mt. Everest, how much slower could he run to stay at Mach .02?



## Mathematics with the X-15

### Exercise 1 Key

- 1:  $\frac{354,200 \text{ feet}}{5,280 \text{ feet/mile}} = 67.08 \text{ miles}$
- 2:  $354,200 \text{ feet} - 45,000 \text{ feet} = 309,200 \text{ feet}$
- 3:  $\frac{309,000 \text{ feet}}{5,280 \text{ feet/mile}} = 58.6 \text{ miles}$
- 4: *answers will vary*
- 5:  $4,520 \text{ miles/hour} - 4,486 \text{ miles/hour} = 34 \text{ miles/hour}$
- 6:  $\frac{4,520 \text{ miles per hour}}{65 \text{ miles per hour}} = 69.5 \text{ times faster!}$
- 7:  $762 \text{ miles/hour} - 87 \text{ miles/hour} = 675 \text{ miles/hour}$
- 8:  $\frac{4,520 \text{ miles hour}}{675 \text{ miles/hour}} = \text{Mach } 6.7$
- 9: *6.7 times the speed of sound*

### Exercise 2 Key

- 1:  $\frac{65 \text{ miles/hour}}{762 \text{ miles/hour}^*} = \text{Mach } .085$   
*\*from table*
- 2: *from table: for altitude range of 20,000 - 30,000 feet, the speed of sound in the stated air conditions is 693 miles/hour. When the pilot is flying at Mach 1, he/she is flying at the speed of sound, or 693 miles/hour.*
- 3:  $762 \text{ miles/hour at sea level} - 693 \text{ miles/hour at } 29,028 \text{ feet} = 69 \text{ miles/hour}$
- 4:  $354,200 \text{ feet} - 29,028 \text{ feet} = 325,172 \text{ feet}$   
 $325,172 \text{ feet} / 5,280 \text{ feet} = 61.6 \text{ miles}$   
*answers will vary*
- 5:  $\frac{3,111 \text{ miles/hour}}{662 \text{ miles/hour}} = \text{Mach } 4.7$
- 6:  $\frac{15 \text{ miles per hour}}{762 \text{ miles per hour}} = \text{Mach } .02$   
*That's two one-hundredths of the speed of sound!*  
 $\text{Mach } .02 \times 693 \text{ miles/hour} = 13.86 \text{ miles/hour}$



## Don't Let It Weigh You Down!

**Background:** When flying in a lighter-than-air balloon, the load you carry cannot weigh more than what the balloon can carry. Many years ago, through trial and error, methods were developed to accurately predict how much weight a balloon could carry. These methods use four measurements:

- the size of the inside of the balloon and how much gas it can carry - called the "volume" of the balloon
- how much the equipment weighs (including the balloon itself, ropes, gondola, and the gas)
- how much the aeronaut and passengers weigh
- the density of the gas

Say that you are an aeronaut who had planned to take your three cousins on a balloon ride to see the countryside from the air. You also planned to bring a picnic lunch to feed everyone (including yourself!) and, since it is cooler up in the air, you wanted to bring some blankets to keep everyone warm. Just when you had everything ready to go, your cousin from far away paid a surprise visit and wanted to go along also. Given all the facts below, can your cousin go along?

In preparation for the flight, you had calculated the total weight of all the people and equipment you expected to bring along. Your calculations were as follows:

Item	Weight
You (the aeronaut)	80 pounds
Cousin Susanne	65 pounds
Cousin Phil	70 pounds
Cousin Andrew	75 pounds
balloon	250 pounds
gondola	300 pounds
ropes and other equipment	50 pounds
lunch for four people	20 pounds
blankets for four people	8 pounds
<b>Total</b>	<b>918 pounds</b>



Your balloon, with brilliant red, white and blue stripes, is as tall as a three-story building and can carry 89,000 cubic feet of gas. You can say that the volume of your balloon is 89,000 cubic feet. You also know from your study of chemistry that the density of helium is .011 pounds per cubic foot.

To calculate how many pounds your balloon could carry, you multiplied the density of the helium by the volume of the balloon.

Density of the helium = .011 pounds per cubic foot

Volume of the balloon = 89,000 cubic feet

.011 pounds per cubic foot X 89,000 cubic feet = 979 pounds

So, based on your calculations, you could carry 979 pounds on your flight. On the list above, where you totaled the weight of all the items you expected to carry, you expected to carry 918 pounds.

Would you be able to fly your three cousins, plus lunch and blankets on your balloon? Yes, because they weighed 918 pounds and you could carry 979!

But what about your cousin Bryant who wanted to come along? Bryant tells you that he weighs 85 pounds. You add Bryant's weight to the total weight of all the items you expected to carry:

918 pounds + 85 pounds = 1,003 pounds

Oh, no! Bryant cannot fly with you! Can you tell why?

That's right! The reason is that with the addition of Bryant the total weight of the items you want to carry becomes too big for the balloon you have.

What can you do so Bryant can go along? Well, you calculated that your balloon could carry 979 pounds. How much over that limit are you if Bryant comes along?

1,003 pounds - 979 pounds = 24 pounds

So, you must remove 24 pounds from your weight list. Obviously you cannot remove the people, the gondola, the balloon or the ropes and other equipment. What's left?



The total weight of the lunch and blankets is:

$$20 \text{ pounds} + 8 \text{ pounds} = 28 \text{ pounds}$$

So, if you left the lunch and blankets at home, the total weight you need to carry is:

$$1,003 \text{ pounds} - 28 \text{ pounds} = 975 \text{ pounds}$$

Without the lunch and blankets the total weight of 975 pounds is less than the 979-pound limit that your balloon can carry. You need to make a decision! Bryant may come along on the flight and the lunch and blankets stay home, or the lunch and blankets come along and Bryant stays home. What will you do?

Use the following template to help you complete the exercises below.



## Don't Let It Weigh You Down!

### Template

**Step 1:** Fill in the following weight list table. List the items you want to take along in the left-hand column, and the weight of each item in the right-hand column.

Item	Weight
Total	





## Exercises

- 1:** Congratulations! You have just bought a brand new balloon! Your new balloon weighs 200 pounds, the gondola weighs 300 pounds and the ropes and other equipment, 75 pounds. You'd like to take your Aunt and Uncle along for a ride. Your Aunt weighs 110 pounds and your Uncle weighs 160 pounds. You weigh 90 pounds. Your Aunt's favorite candy is chocolate truffles, so you want to bring along a 5-pound box of candy for her. The volume of your balloon is 86,000 cubic feet. Can your balloon carry all the people, equipment and candy you want to carry?
- 2:** The National Weather Service (NWS) often launches balloons to carry instruments into the atmosphere. One day, during mid-August, radar detects a huge storm brewing. The NWS needs to send a balloon up to help them figure out if the storm is really going to be as big as it appears. The balloon will not have any passengers - only instruments will be on board. The instruments weigh 335 pounds. The gondola weighs 105 pounds. Are there any other weights that you need? If so, go back to the very first example problem and choose what you need from the weight table. Next, decide what size balloon you need to launch the instruments.
- 3:** You and a friend are going to give a birthday party for one of your classmates. You want to decorate with lots of balloons. You go to the store and buy a package of 100 different colored balloons. The volume of each balloon in the package is 9 cubic feet. How many pounds of helium must you buy to fill up all the balloons?

If the helium comes in 3-pound canisters, how many canisters must you buy?



## Exercises Key

- 1: Congratulations! You have just bought a brand new balloon! Your new balloon weighs 200 pounds, the gondola weighs 300 pounds and the ropes and other equipment, 75 pounds. You'd like to take your Aunt and Uncle along for a ride. Your Aunt weighs 110 pounds and your Uncle weighs 160 pounds. You weigh 90 pounds. Your Aunt's favorite candy is chocolate truffles, so you want to bring along a 5-pound box of candy for her. The volume of your balloon is 86,000 cubic feet. Can your balloon carry all the people, equipment and candy you want to carry?

*weight of all people, etc. = 940 pounds*  
*.011 lbs/cubic foot X 86,000 cubic feet = 946 lbs*  
*your balloon can carry everything*

- 2: The National Weather Service (NWS) often launches balloons to carry instruments into the atmosphere. One day, during mid-August, radar detects a huge storm brewing. The NWS needs to send a balloon up to help them figure out if the storm is really going to be as big as it appears. The balloon will not have any passengers - only instruments will be on board. The instruments weigh 335 pounds. The gondola weighs 105 pounds. Are there any other weights that you need? If so, go back to the very first example problem and choose what you need from the weight table. Next, decide what size balloon you need to launch the instruments.

*from example table: balloon = 250 lbs; ropes, etc. = 50 lbs*  
*total weight = 740 lbs*  
*740 lbs / .011 lbs/cubic foot = 67,273 cubic feet*

- 3: You and a friend are going to give a birthday party for one of your classmates. You want to decorate with lots of balloons. You go to the store and buy a package of 100 different colored balloons. The volume of each balloon in the package is 9 cubic feet. How many pounds of helium must you buy to fill up all the balloons?

*.011 lbs per cubic foot X 900 cubic feet = 9.9 lbs*

If the helium comes in 3-pound canisters, how many canisters must you buy?

*9.9 lbs / 3 lbs = 3.3 canisters - so you need to buy 4.*



## The Aspect Ratio of Wings

**Review:** As air flows over and under a wing, we know from our study of lift that the air flowing over the top flows faster than the air that flows under the wing. We also know from Bernoulli's Principle that the air that flows faster applies less pressure to the surface it is flowing over. Therefore, since the air flowing over the top of a wing has less pressure (because it is flowing faster), the air pressure on top is less than on the bottom of the wing. The higher air pressure on the bottom "lifts" the wing.

**Background:** When engineers design a new airplane, the size and shape of the wings are a very important issue. Wings provide the majority of the lift for the airplane, but they also cause drag. Remember that drag is a force that opposes the thrust force. Engineers are always trying to find ways to increase lift and reduce drag caused by the wings.

In addition to flowing faster, the air that flows over the top of the wing also tends to flow inward, toward the fuselage. The air that flows over the bottom is flowing more slowly and tends to flow outward. As these two airflows meet along the trailing edge of the wing, they form a rotating column of air that extends from the wing tip. This is called a wing-tip vortex.

If they are lucky, passengers riding behind the wing of an airplane can sometimes see a wing-tip vortex - particularly if they are flying in the morning or on a slightly humid day. It looks like a long, slim whirlwind that extends from the tip of the wing.

Unfortunately, while they are fun to watch, the same characteristics of the airflow that create wing-tip vortices (the plural of vortex is vortices) also create drag.



## Teacher - Led Exercise

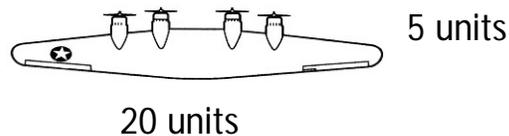
**Directions:** In their efforts to increase lift and reduce drag, engineers use a mathematical formula called the "aspect ratio". The "aspect ratio" is simply a comparison between the length and width of the wing:

$$\frac{\text{length of the wing}}{\text{width of the wing}} = \text{aspect ratio}$$

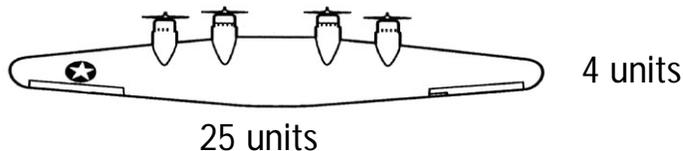
Experiments have shown that a wing built with a higher aspect ratio tends to create less drag than a wing built with a smaller aspect ratio - even when their area remains the same!

Examine the three wings drawn below, calculate the area and aspect ratio of each wing, and fill in the following table. Then, rank the wings according to the drag that each will create, given their aspect ratios. Rank the wing with the least drag, number 1 and the greatest amount of drag, number 3.

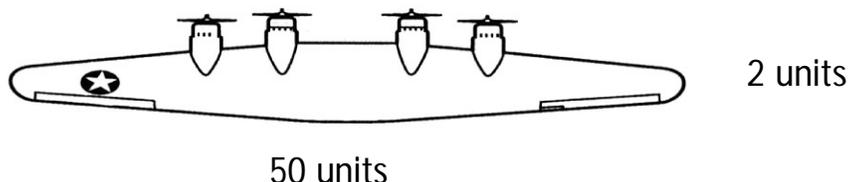
**Wing "A"**



**Wing "B"**



**Wing "C"**





<b>Wing</b>	<b>length</b>	<b>width</b>	<b>area</b>	<b>aspect ratio</b>	<b>drag ranking</b>
<b>A</b>					
<b>B</b>					
<b>C</b>					



## Exercise 2

**Step 1:** Create and draw your own wings below. Shape them like airfoils and give each the same area of 200 square units.

**Wing A**

**Wing B**

**Step 2:** Label the length and width of each wing.

**Step 3:** Calculate the aspect ratio for each wing and fill in the table below. Don't forget to include the units!

Wing	length	width	area	aspect ratio	drag ranking
A					
B					

**Step 4:** Rank the wings according to the drag that each will create, given their aspect ratios. Rank the wing with the least drag, number 1 and the one with the greatest amount of drag, number 2.



## The Aspect Ratio of Wings

### Teacher - Led Exercise Key

**Wing "A":** length: 20 units      width: 5 units

**Wing "B":** length: 25 units      width: 4 units

**Wing "C":** length: 50 units      width: 2 units

<b>Wing</b>	<b>length</b>	<b>width</b>	<b>area</b>	<b>aspect ratio</b>	<b>drag ranking</b>
<b>A</b>	20 units	5 units	100 square units	4	3
<b>B</b>	25 units	4 units	100 square units	6 R1	2
<b>C</b>	50 units	2 units	100 square units	25	1

Even though each wing has the same area, 100 square units, Wing "C" has the greatest aspect ratio, and Wing "A" has the smallest aspect ratio. This implies that Wing "A" creates more drag than Wing "C".

Maybe you've wondered why sailplanes and gliders have long, slim wings. Since they don't have engines to provide thrust, their wing shape helps to provide the greatest amount of lift with the least amount of drag. Check out the ER-2 in [The Hangar](#) section of the CD-ROM. The ER-2 is a rocket-powered glider!



## Exercise 1 Key

**Step 1:** Possible wing dimensions and aspect ratios:

<i>length = 9</i>	<i>width = 8</i>	<i>aspect ratio = 1 R1</i>
<i>length = 12</i>	<i>width = 6</i>	<i>aspect ratio = 2</i>
<i>length = 36</i>	<i>width = 2</i>	<i>aspect ratio = 18</i>
<i>length = 24</i>	<i>width = 3</i>	<i>aspect ratio = 8</i>
<i>length = 18</i>	<i>width = 4</i>	<i>aspect ratio = 4 R2</i>

<b>Wing</b>	<b>length</b>	<b>width</b>	<b>area</b>	<b>aspect ratio</b>	<b>drag ranking</b>
<b>A</b>	<i>9 units</i>	<i>8 units</i>	<i>72 square units</i>	<i>1R1</i>	<i>2</i>
<b>B</b>	<i>12 units</i>	<i>6 units</i>	<i>72 square units</i>	<i>2</i>	<i>1</i>

## Exercise 2 Key

**Step 1:** Possible wing dimensions and aspect ratios:

<i>length = 100</i>	<i>width = 2</i>	<i>aspect ratio = 50</i>
<i>length = 50</i>	<i>width = 4</i>	<i>aspect ratio = 12 R2</i>
<i>length = 20</i>	<i>width = 10</i>	<i>aspect ratio = 2</i>
<i>length = 25</i>	<i>width = 8</i>	<i>aspect ratio = 3 R1</i>

<b>Wing</b>	<b>length</b>	<b>width</b>	<b>area</b>	<b>aspect ratio</b>	<b>drag ranking</b>
<b>A</b>	<i>100 units</i>	<i>2 units</i>	<i>200 square units</i>	<i>50</i>	<i>1</i>
<b>B</b>	<i>20 units</i>	<i>10 units</i>	<i>200 square units</i>	<i>2</i>	<i>2</i>



## Computing the Net Force

**Review:** One way to start a class dialog on “force” is to ask students to give examples from their own experience of a “force”. Responses might include a “hit” or some sort of forceful contact; others might be more group-oriented, like the “Air Force”; another possibility is “The Force” from the Star Wars movies. There are very few wrong answers to this question, and some reflection on their own experiences often helps students when they try to grasp the slightly more formal definition below.

A force is defined in its simplest sense as a “push” or a “pull”. These definitions do not imply a direction. Students can “pull” in any direction as they can “push” in any direction! The terms are frequently used because students can readily identify with the actions of pushing and pulling, and the fact that these actions usually have an effect on what they are pushing or pulling.

Review with students that there are two parts to the definition of a force. In fact, when a force is defined it must have both parts - one is not enough! The two parts are: magnitude (a quantity that can be measured) and direction. The direction of a force is self-explanatory, and again, has nothing to do with the terms “push” or “pull”.

The magnitude of a force can be described as “how hard the force is”, or “how much power the force has”. For example, a force of magnitude 10 can be described as a “stronger” force than one of magnitude 2, which can be described as a “weaker” force.

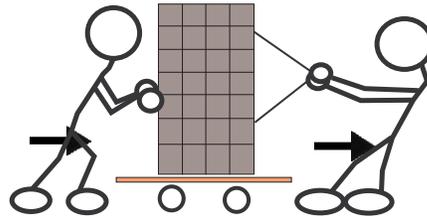
Special note: When working with this lesson, it is very important that students learn to draw accurate pictures of the events described!

**Background:** When two forces act in parallel, in either the same or opposite direction, measuring them is simply a matter of adding or subtracting their magnitudes. When two forces are acting in parallel and in the same direction, measure them by adding the magnitudes together.



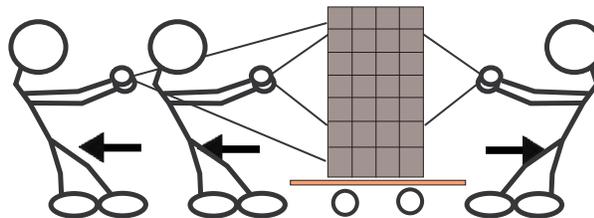
In the example below, a “push” of magnitude 1 added to a “pull” of magnitude 1 equals a net force of magnitude 2. The cart will then move in the direction of the greatest magnitude - in this case to the right.

**Push 1 + Pull 1 = Net Force 2 to the right**



When two forces act in parallel in the opposite direction, measure them by subtracting the magnitudes. In the example below, a pull of magnitude 1 is acting opposite to a pull of magnitude 2. The cart will move in whichever direction has the greatest magnitude. In this case the cart will move to the left.

**Push 2 - Pull 1 = Net Force 1 to the left**



You may want to walk the students through a similar process using ropes or string and students of equal size to demonstrate the concept.

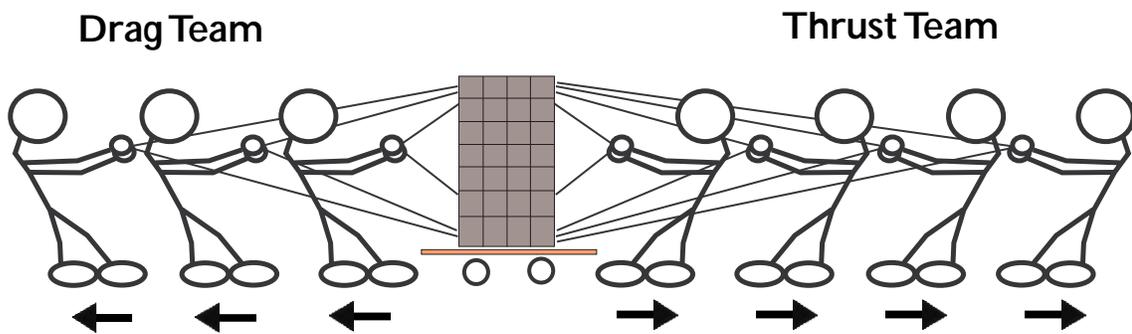
Forces that act in opposite directions are called “oppositional” forces. Four of the forces in aeronautics (lift, drag, weight, and thrust) can be thought of as “oppositional” pairs.

*thrust acts in a direction opposite to drag*

*lift acts in a direction opposite to weight*



The oppositional forces can be introduced as a game of tug-of-war. Teams can be named as the four forces. For example, a tug-of-war can be set up between a “thrust” team and a “drag” team.



In the above graphic, the “Thrust Team” has a magnitude of 4 and the “Drag Team” has a magnitude of 3. The net force will be

$$\text{Thrust } 4 - \text{Drag } 3 = \text{Net Force } 1 \text{ to the right}$$

Since the “Thrust Team” has the greater magnitude, the cart will move in the direction that the “Thrust Team” is pulling, in this case to the right.



## Worksheet

**Question 1:** Define the word "force".

**Question 2:** Complete the sentences below by filling in the blanks.

A force can move in different \_\_\_\_\_.

A force has "strength" or \_\_\_\_\_ that can be \_\_\_\_\_.

Parallel forces can be added or \_\_\_\_\_.

**Question 3:** An F-14 is flying west. Its engines are creating a thrust force of magnitude 4,000. A strong headwind is blowing to the east creating a drag force on the F-14 of magnitude 1,000.

What is the net force on the F-14? \_\_\_\_\_

In what direction will the F-14 fly? \_\_\_\_\_

Draw a picture of this event. Make sure you include the F-14, the wind, arrows to represent the magnitudes, and the equation that gives the net force. Draw one arrow for each 1,000 units of magnitude.

**Question 4:** After the Space Shuttle is launched, its huge rocket engines lift it upward with incredible force. As it blasts through the top of the atmosphere into outer space, the engines are creating a force pushing up into space with a magnitude of 6 times the force of gravity. We write this as "6g".

The gravity force is pulling the Shuttle back down in the direction of the earth with a magnitude of 1 times the force of gravity. We write this as "1g".

What is the net force on the Space Shuttle? \_\_\_\_\_

Draw a picture of this event to help you answer the question. Be sure



to include the Shuttle, the Earth, arrows to represent which direction the engines and the earth are pulling, and the equation that gives the net force. Draw one arrow for each g.

**Question 5:** Four people are pulling on ropes attached to a cart. Each person is pulling with a magnitude of 1. Two people are pulling to the right and two people are pulling to the left.

What is the magnitude of the net force? \_\_\_\_\_

In which direction will the cart move? \_\_\_\_\_

Draw a picture of this event to help you answer the questions. Be sure to include the cart, the people, arrows to represent the directions that the people are pulling, and the equation that gives the net force. Draw one arrow for each unit of magnitude.



## Worksheet Key

**Question 1:** *A force is a "push" or a "pull". It has two parts: magnitude and direction.*

**Question 2:** *directions  
magnitude, measured  
subtracted*

**Question 3:** *3,000  
West  
thrust 4,000 - drag 1,000 = net force 3,000 in the direction of thrust*

**Question 4:** *5g  
up 6g - down 1g = net force 5g in up direction **or**  
lift 6g - weight 1g = net force 5g in direction of lift*

**Question 5:** *0  
neither  
pull 1 - pull 1 = net force 0*



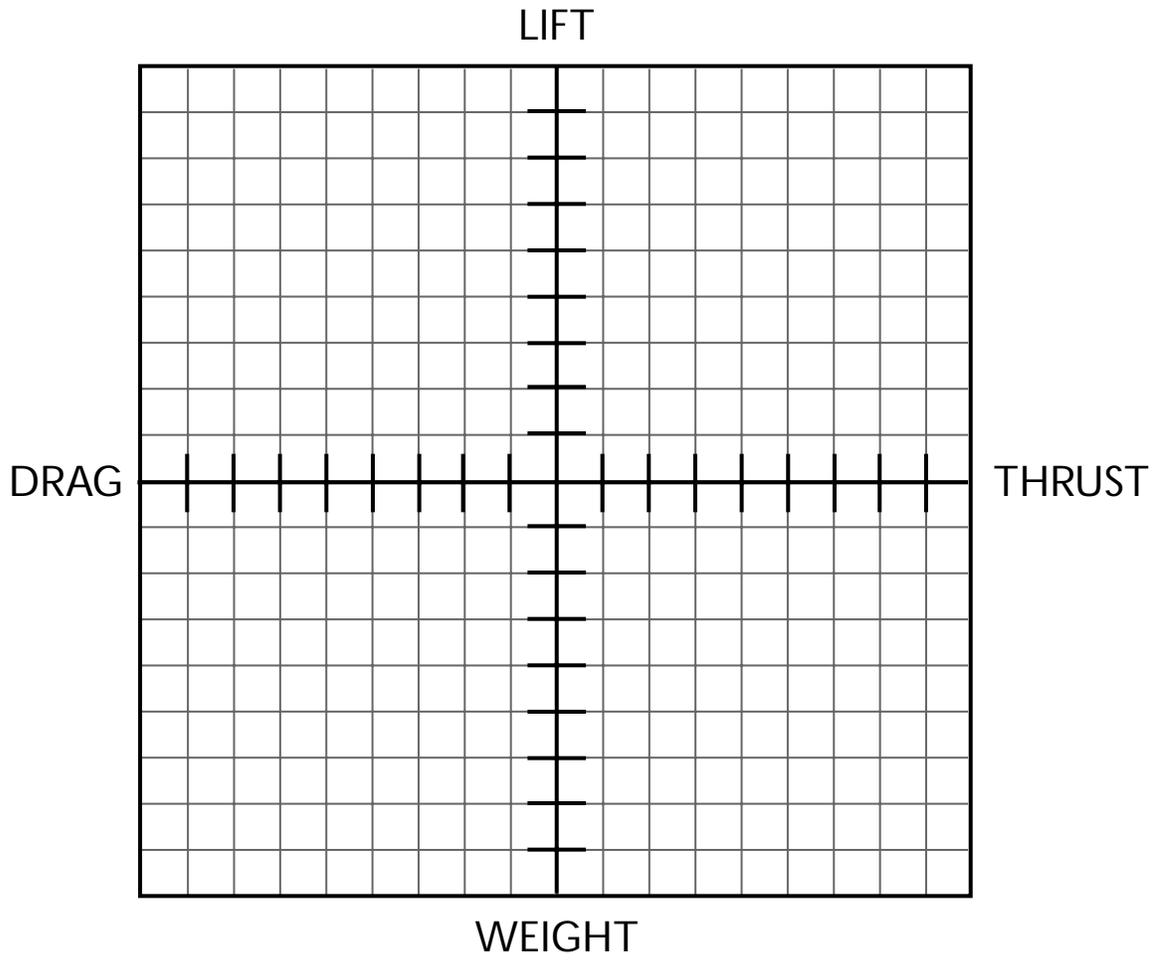
## Graphing the Four Forces

**Background:** The concept of force can be effectively represented on a graph using the Cartesian coordinate system. By representing four of the aeronautical forces (lift, drag, thrust, weight) on a graph, students can visualize both parts of the definition of force: magnitude and direction.

In the lesson, Computing the Net Force, students learned how to calculate the magnitude and direction of the net force, given two parallel forces. In this lesson, students will use information about four forces to make a decision about whether or not an airplane is (theoretically!) able to fly.

This lesson concentrates on the actual representation of the forces on a graph. If, after combining the four forces, the net force is plotted in the upper right quadrant of the graph, then we will draw the conclusion that the airplane is able to fly.

**Directions:** Have students examine the graph on the following page. Point out that lift is "up toward the top of the paper", weight is "down toward the bottom of the paper", thrust is "forward toward the right of the paper" and drag is "back toward the left of the paper".





Using the magnitudes below, follow the steps and plot your points on the graph on the previous page.

Weight	3 units
Lift	7 units
Drag	2 units
Thrust	5 units

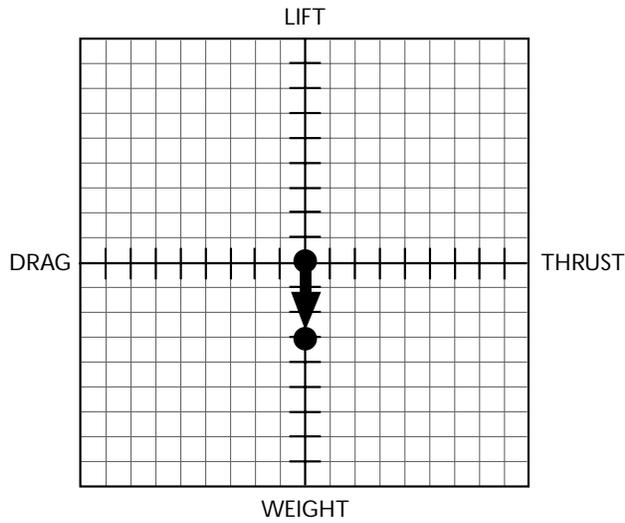
The forces can be plotted in any order. For example:

- Step 1:** Start at the origin and count down three squares (for Weight). Plot a small dot.
- Step 2:** From that small dot (do not start again from the origin!) count up seven squares (for Lift). Plot another small dot.
- Step 3:** From that small dot (do not start again from the origin!) count to the left two squares (for Drag). Plot another dot.
- Step 4:** From that small dot (do not start again from the origin!) count to the right 5 squares (for Thrust). Plot a large dot. This is the representation of the net force.

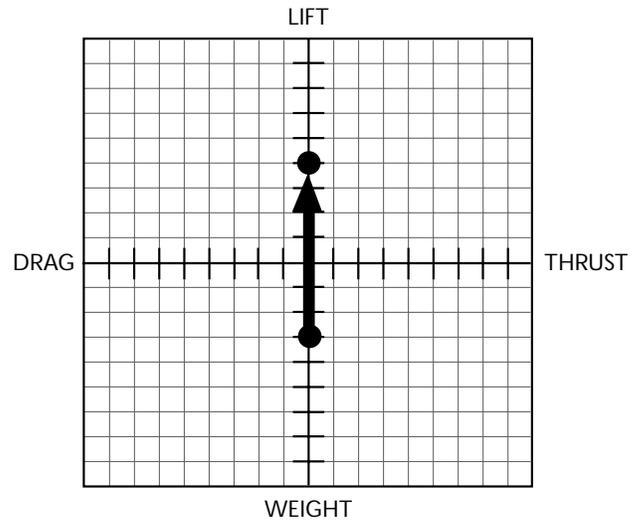
See the graphs on the following page for guidance, then continue.



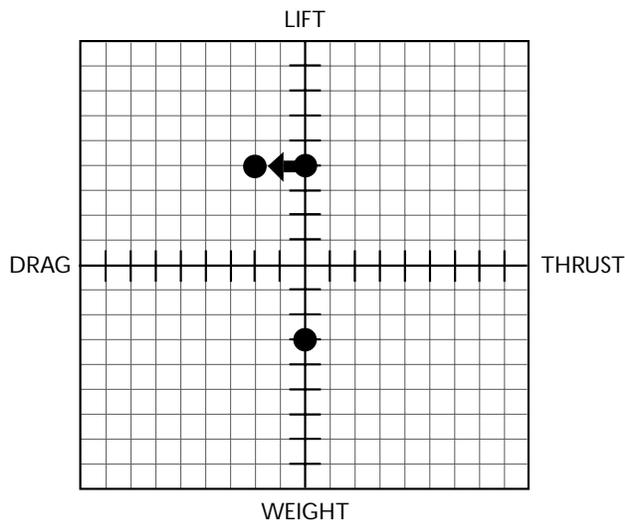
### Step 1: Down 3



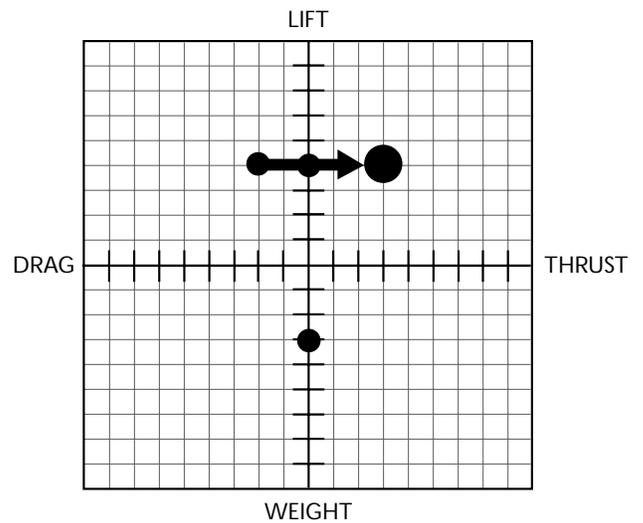
### Step 2: Up 7



### Step 3: Left 2



### Step 4: Right 5





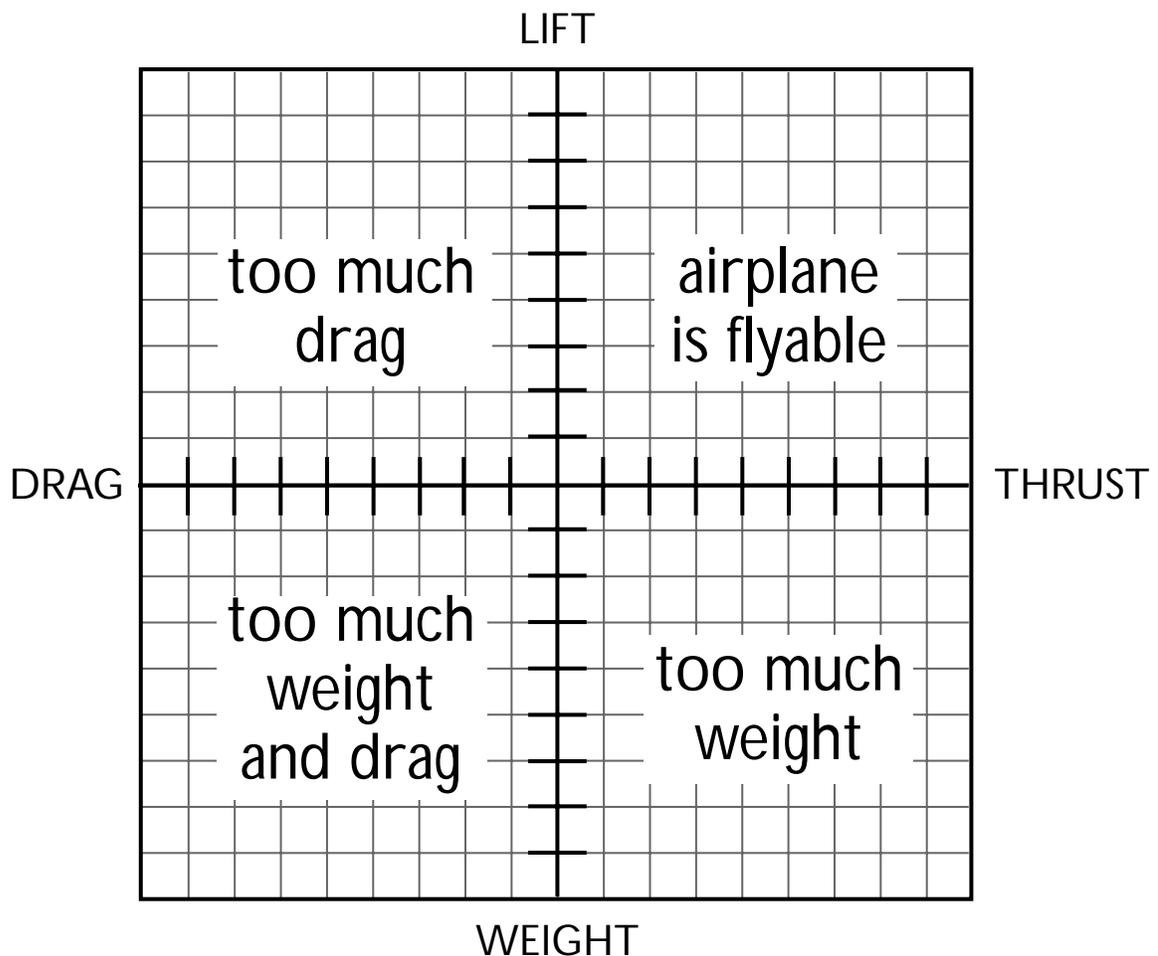
**Step 5:** Determine whether or not the airplane is flyable.

If the net force is plotted in the upper right quadrant, the airplane is flyable.

If the net force is plotted in the upper left quadrant, the airplane is not flyable - it has too much drag.

If the net force is plotted in the lower left quadrant, the airplane is not flyable - it has too much drag and weight.

If the net force is plotted in the lower right quadrant, the airplane is not flyable - it has too much weight.

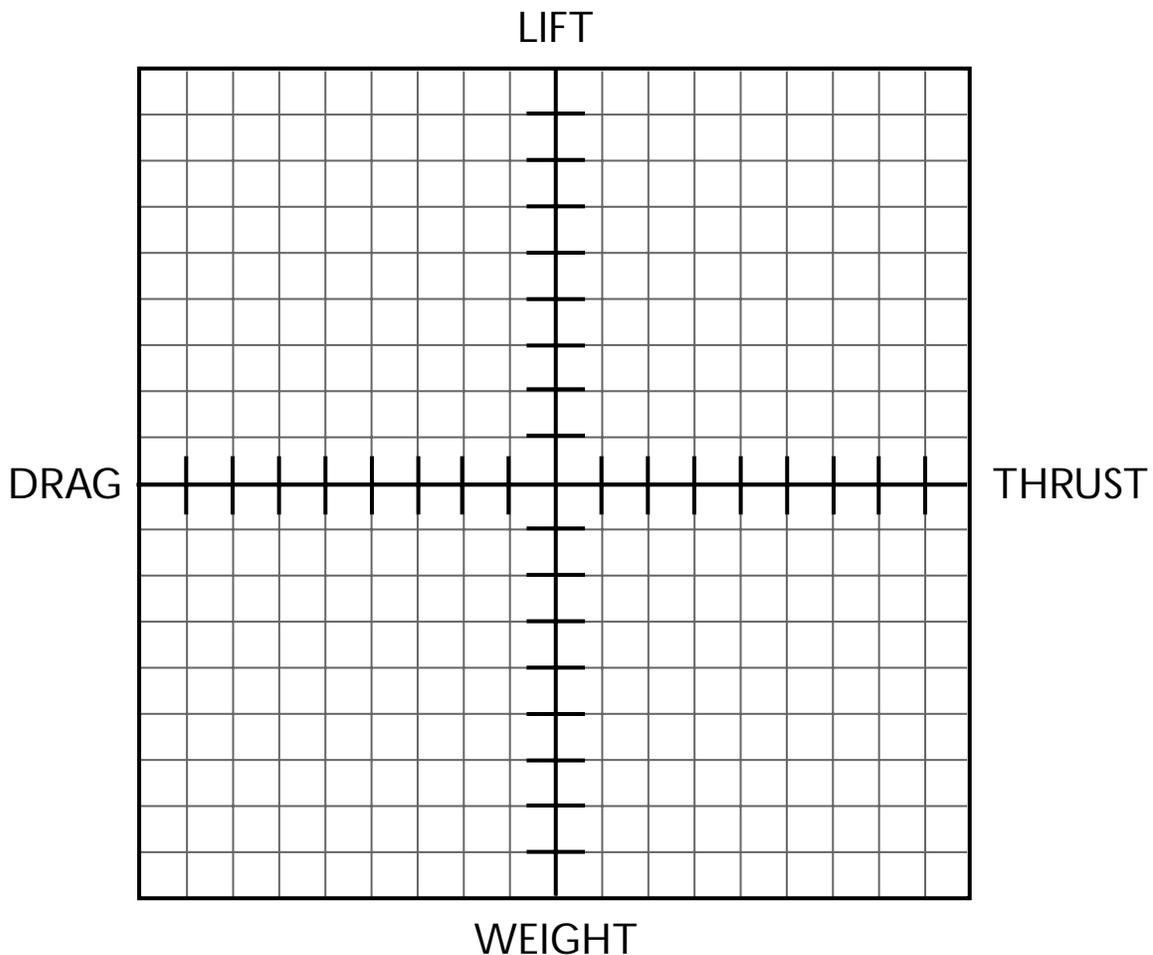




## Exercise 1

**Directions:** Use the steps from the previous example to plot the following magnitudes. After you plot the net force, make a decision about whether or not the airplane is flyable.

Weight	4 units
Lift	10 units
Drag	2 units
Thrust	10 units



**Question 1:** This plane is / is not flyable.

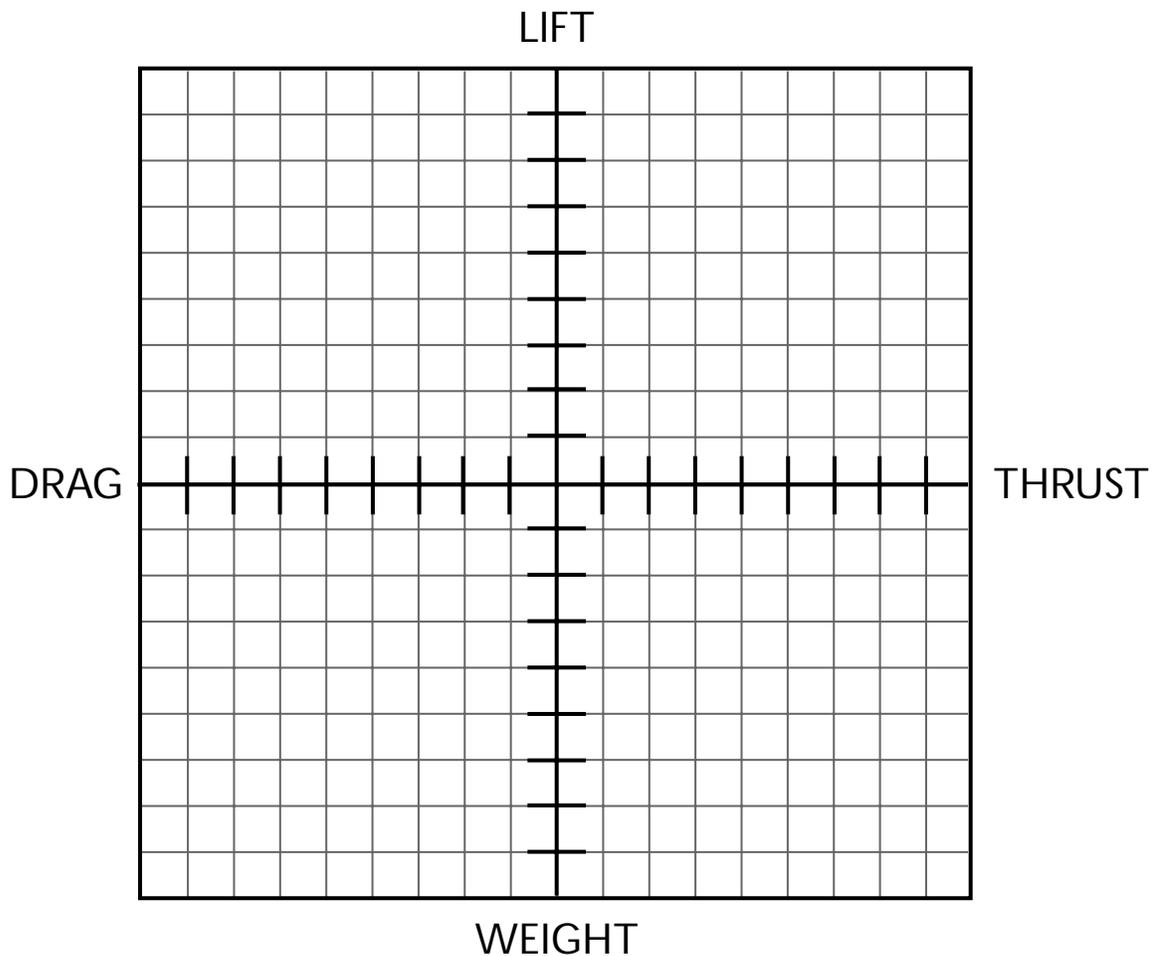
**Question 2:** If it is not, tell what force or forces are too great for the airplane to fly.



## Exercise 2

**Directions:** Use the steps from the previous example to plot the following magnitudes. After you plot the net force, make a decision about whether or not the airplane is flyable.

Weight	8 units
Lift	4 units
Drag	6 units
Thrust	4 units



**Question 1:** This plane is / is not flyable.

**Question 2:** If it is not, tell what force or forces are too great for the airplane to fly.



## Graphing the Four Forces

### Exercise 1 - Key

*Starting at the origin:*

*the end of the weight arrow will be at (0,-4)*

*the end of the lift arrow will be at (0,6)*

*the end of the drag arrow will be at (-2,6)*

*the end of the thrust arrow will be at (8,6)*

*since (8,6) is in the upper right quadrant, the airplane is flyable*

*Note: the arrows may be drawn in any order, you will always end up at the same place!*

### Exercise 2 - Key

*Starting at the origin:*

*the end of the weight arrow will be at (0,-8)*

*the end of the lift arrow will be at (0,-4)*

*the end of the drag arrow will be at (-6,-4)*

*the end of the thrust arrow will be at (-2,-4)*

*since (-2,-4) is in the lower left quadrant, the airplane is not flyable*

*both weight and drag are too great*



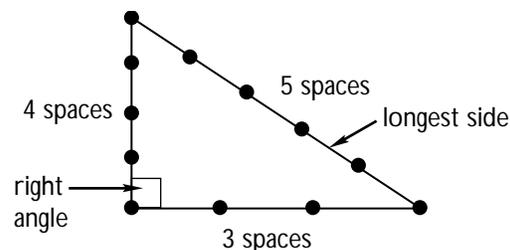
## Flying With Pythagoras

**Preparation:** Students should be familiar with the concepts of squares and square roots. They should be able to use their calculators to square numbers and find the square roots of numbers.

**Background:** In the 6th century BC, a Greek philosopher named Pythagoras lived in the village of Samos. He started a school where philosophy and religion were studied, in addition to astronomy, mathematics and music. The students from his school were called Pythagoreans. Central to Pythagoras' teaching was the idea that all physical relationships could be expressed by mathematical relations. One of the most famous discoveries of the Pythagoreans was a proof for a distance relationship that had been developed many years before by the Egyptians.

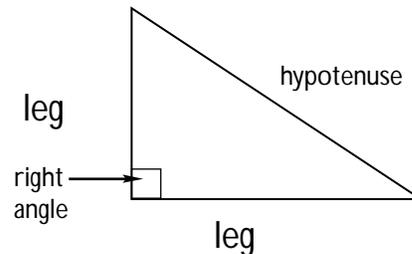
The Nile River flows through Egypt. This huge river is a source of life in an otherwise barren, desert land. During the rainy season, the Nile floods regularly. After each flood the surveyors would have to reset the boundaries of the farmers' fields. Land was sectioned into squares, so it was critical that the surveyors knew how to mark a right angle (because squares have four right angles).

The clever Egyptians took a rope and tied twelve evenly-spaced knots in it. They then made a triangle with the rope. One side had three spaces between the knots, another had four spaces between the knots, and the longest had five. This triangle was very special. The angle opposite from the longest side was always a right angle. Using this rope, the surveyors were able to show that the boundaries they marked were indeed in the shape of a square.





Many years later, the Pythagoreans named a triangle that contained a right angle, a "right triangle". They also named some of the parts of a right triangle. They called the longest side, opposite the right angle, the hypotenuse. The sides next to (or adjacent to) the right angle were called the legs.



The Pythagoreans discovered that the legs and hypotenuse of a right triangle did not always have to have lengths of 3, 4 and 5. But the numbers did have to work in a special formula. The special formula is called the Pythagorean Theorem. The Pythagorean Theorem goes like this:

If you take the length of a leg of a triangle (say "a" in the graphic above) and multiply it by itself (or "square" it);

$$a \times a \quad \text{or} \quad a^2$$

then, do the same with the length of the other leg ("b" in the graphic above);

$$b \times b \quad \text{or} \quad b^2$$

and add the results together;

$$a^2 + b^2 =$$

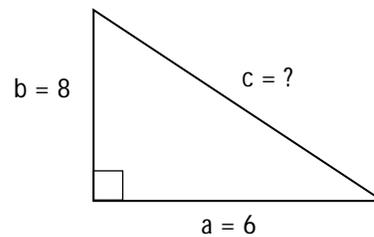
your final result will be equal to the length of the hypotenuse ("c" in the graphic above) multiplied by itself (or "squared").

$$a^2 + b^2 = c^2$$



The Pythagoreans also discovered that if they knew the lengths of the two legs of a right triangle, they could use the Pythagorean Theorem to find the length of the hypotenuse.

Say that one leg of a right triangle has a length of 6 units and another has a length of 8 units. What is the length of the hypotenuse?



We know from the Pythagorean Theorem that

$$a^2 + b^2 = c^2$$

In our example,  $a = 6$  and  $b = 8$ . So,

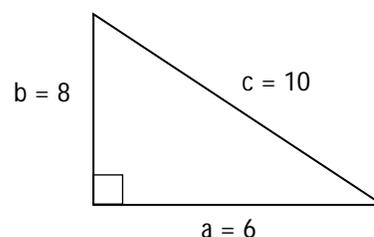
$$a^2 + b^2 = c^2$$

$$6^2 + 8^2 = c^2$$

$$36 + 64 = c^2$$

$$100 = c^2$$

Since the square root of 100 is 10 (that is,  $10^2$  equals 100) the length of the hypotenuse must equal 10.





## Exercises

**Directions:** Use the information given and the Pythagorean Theorem to solve the following problems.

**Problem 1:** The length of leg “a” of a right triangle is 9, the length of leg “b” is 12. What is the length of the hypotenuse?

Draw the triangle. Make sure you mark the lengths of the two legs and the hypotenuse, and tell which angle is the right angle.



**Problem 2:** NASA Test Pilot Loren Haworth is instructed to fly the following mission in a brand new aircraft, the X-99. He will be testing the aircraft's ability to follow a flight path very precisely. Test Pilot Haworth is instructed to fly North from San Antonio, Texas to Sioux Falls, South Dakota, a distance of 1,000 miles. He is instructed to then fly east to Scranton, Pennsylvania, a distance of 1,200 miles. After reaching Scranton, he is supposed to fly directly back to San Antonio. What is the distance he must fly from Scranton to San Antonio? Hint: Draw his route and label the distances on the map below to help you find the return distance.

Sioux Falls



Scranton



San Antonio





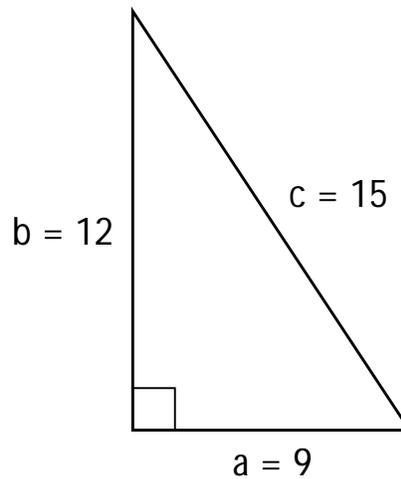
**Problem 3:** One day the Space Shuttle blasts off from the launch pad at the Kennedy Space Center in Florida. Unfortunately, a computer malfunctions and, after reaching an altitude of 4 miles, the Shuttle must return to earth and land on a runway. The runway is 5 miles away from the launch pad. How far must the Shuttle fly from its highest altitude to the runway? Hint: Draw a picture and label the mileages of the Space Shuttle's route to help you find the return distance.



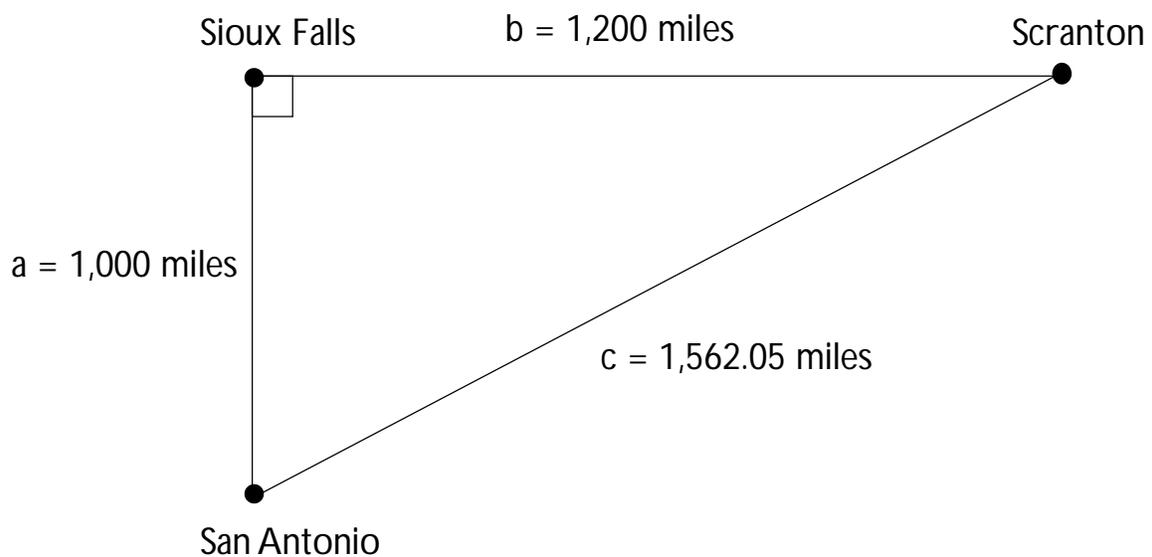
## Flying With Pythagoras

### Exercises Key

- 1: *The length of the hypotenuse is 15.*

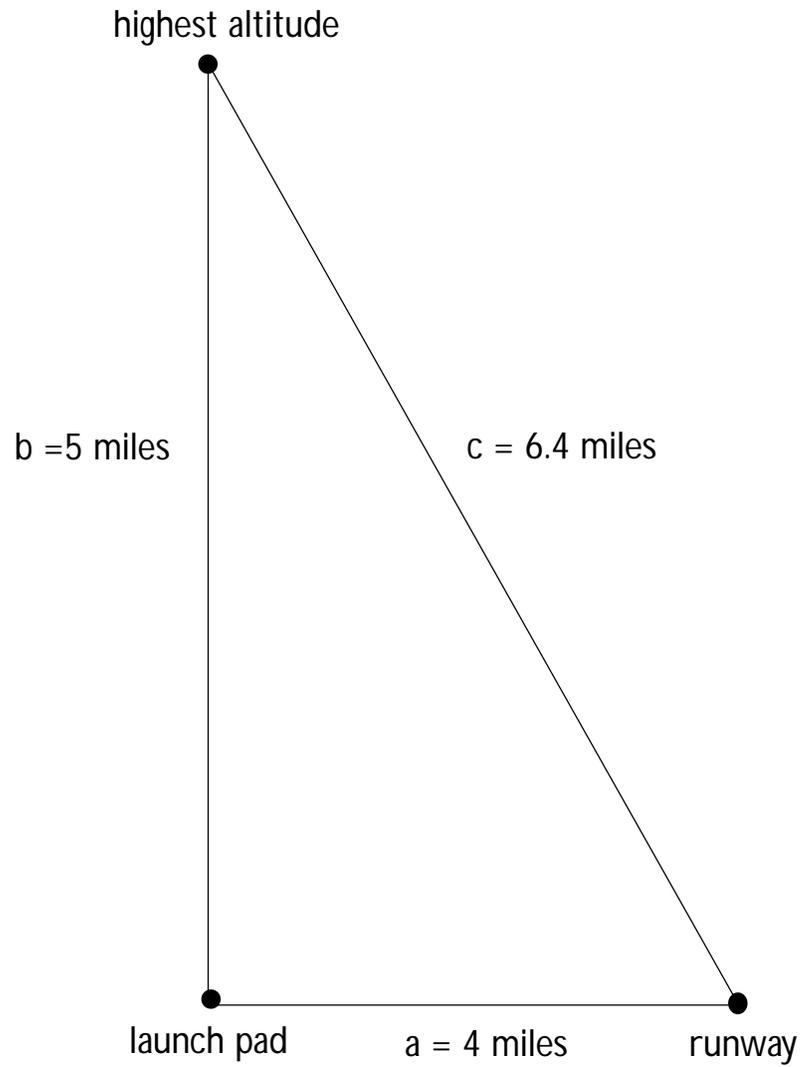


- 2: *The distance from Scranton to San Antonio is 1,562.05 miles.*





3: *The Shuttle must fly 6.4 miles from its highest altitude to the runway.*





## Wind Tunnel Averages

**Background:** When engineers perform wind tunnel tests to measure the forces of drag and lift on a model, they use a unit of measurement called a “newton”. Newtons are named after the famous English physicist, Sir Isaac Newton. A newton is the unit of force it takes to change the velocity of a mass of 1 kilogram, by 1 meter/second over 1 second. Think of a 1 kilogram section of a wing, flying at 250 meters per second. A force of 1 newton would change the velocity of the wing section from 250 meters per second to 251 meters per second, in one second.

If, for instance, a researcher wishes to test the lift experienced by a section of a wing, he or she will embed sensors in various parts of the wing. Each sensor will measure the force of lift on a specific area of the wing. After those values are fed into a computer, the computer will display them in newtons. The researcher can then average all of the values and find the average lift over the entire wing. This same approach can be used for drag.

**Directions:** An average is a way to approximate a value for a large set of numbers. For example, to find the approximate length of the steps you take when you walk, we could measure three, four or ten of your usual walking steps. Then we could average them to find out how long a stride you usually take.

To find an average, follow these two steps:

**Step 1:** Add all of the numbers together.

**Step 2:** Divide the sum by the number of numbers.

The result of this division is the average of the numbers.

For example, let’s say an engineer embedded three sensors in a wind tunnel model to measure the lift force. The computer reported the following values from each sensor:

250 newtons  
300 newtons  
350 newtons



Say that the engineer wanted to find the average of the lift forces over the entire wing. He/She would perform the following steps:

**Step 1:** 250 newtons + 300 newtons + 350 newtons = 900 newtons

**Step 2:**  $\frac{900 \text{ newtons}}{3} = 300 \text{ newtons}$

The average lift force over the entire wing was 300 newtons.



## Exercise

**Directions:** A researcher wanted to find out the quantity of the lift force experienced by different wing types during a wind tunnel test. She embedded three sensors in each of three types of wings: delta, straight, and tapered straight. Her results can be found in the table below. Your task is to find the average lift force for each of the three wing types. Put your answers in the appropriate squares in the table.

### Lift Tests

Wing / Sensor #	1	2	3	Sum	Average
delta	600 newtons	611 newtons	610 newtons		
straight	328 newtons	350 newtons	270 newtons		
tapered straight	390 newtons	433 newtons	440 newtons		

She performed the same experiment again, except that she measured the drag force from the sensors. Her results can be found in the table below. Find the average drag force for each of the three wing types. Put your answers in the appropriate squares in the table.



### Drag Tests

Wing / Sensor #	1	2	3	Sum	Average
delta	26 newtons	32 newtons	23 newtons		
straight	65 newtons	55 newtons	60 newtons		
tapered straight	40 newtons	44 newtons	39 newtons		

**Question 1:** Which wing shows the greatest amount of average lift?

**Question 2:** Which wing shows the least amount of average lift?

**Question 3:** Which wing shows the highest individual lift sensor reading?

**Question 4:** Which wing shows the greatest amount of average drag?

**Question 5:** Which wing shows the lowest individual drag sensor reading?

**Question 6:** If you were to build an airplane, which wing would you use? Why?



## Wind Tunnel Averages

### Exercise 1 Key

#### Lift Tests

Wing / Sensor #	1	2	3	Sum	Average
delta	600 newtons	611 newtons	610 newtons	1821 newtons	607 newtons
straight	328 newtons	350 newtons	270 newtons	948 newtons	316 newtons
tapered straight	390 newtons	433 newtons	440 newtons	1263 newtons	421 newtons

#### Drag Tests

Wing / Sensor #	1	2	3	Sum	Average
delta	26 newtons	32 newtons	23 newtons	81 newtons	27 newtons
straight	65 newtons	55 newtons	60 newtons	180 newtons	60 newtons
tapered straight	40 newtons	44 newtons	39 newtons	123 newtons	41 newtons

**Question 1:** *delta: 607 newtons*

**Question 2:** *straight: 316 newtons*

**Question 3:** *delta: sensor #2 = 611 newtons*

**Question 4:** *straight: 60 newtons*

**Question 5:** *delta: sensor #3 = 23 newtons*

**Question 6:** *The delta because it has the highest lift and the lowest drag. Other answers may be appropriate if the reasoning is good.*



## Graphing Results

**Preparation:** The lesson Wind Tunnel Averages should be completed prior to starting this lesson.

**Background:** When using the four Tools of Aeronautics, engineers create many billions of numbers, which altogether are called data. Wind tunnel tests, flight simulations, Computational Fluid Dynamics and flight tests all produce huge amounts of data. It is very difficult for a human to sift through and analyze millions and millions of numbers. Larger and larger computers have been built to help engineers perform their analysis tasks. One of the fastest modern computers can perform a billion mathematical operations in one second. It would take a human 406 days (without a break!) to do the same task. However, even though the computer can process the massive volumes of data generated by the Tools of Aeronautics, a human engineer is still needed to make decisions based on the data. Computers can display information in many different ways. One of the most effective methods of displaying numerical data is on a graph. Using graphs, engineers can very rapidly analyze and make decisions based upon very large amounts of data.

**Directions:** In this lesson, students will learn how to create a bar graph based on the averages calculated in the lesson Wind Tunnel Averages.

A bar graph has three basic parts:

### **Title**

All bar graphs need a title that tells what kind of data is being shown.

### **Label for Horizontal Axis**

The horizontal axis needs to have a label that identifies the type of data being displayed on that axis (for example, test flights of the X-99).

**Label for Vertical Axis**

The vertical axis needs to have a label that identifies the units of measurement being used (for example, the maximum altitude reached during a test flight)

**Scale for Vertical Axis**

The vertical axis needs to have a scale that lists the units of the measurement used (for example, one mark equals 5,000 feet)

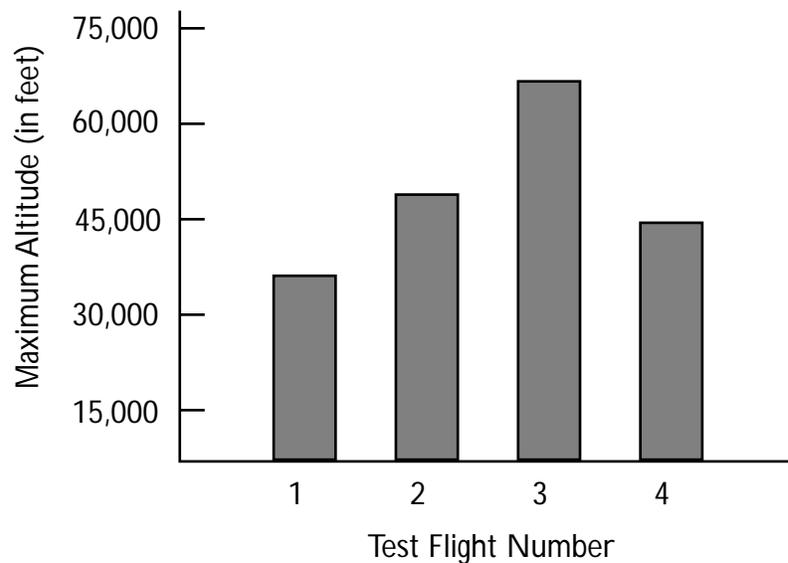
The following information has been used for the graph below.

*Title - "X-99 Flight Test Results"*

*Label for Horizontal Axis - "Test Flight Number"*

*Label for Vertical Axis - "Maximum Altitude"*

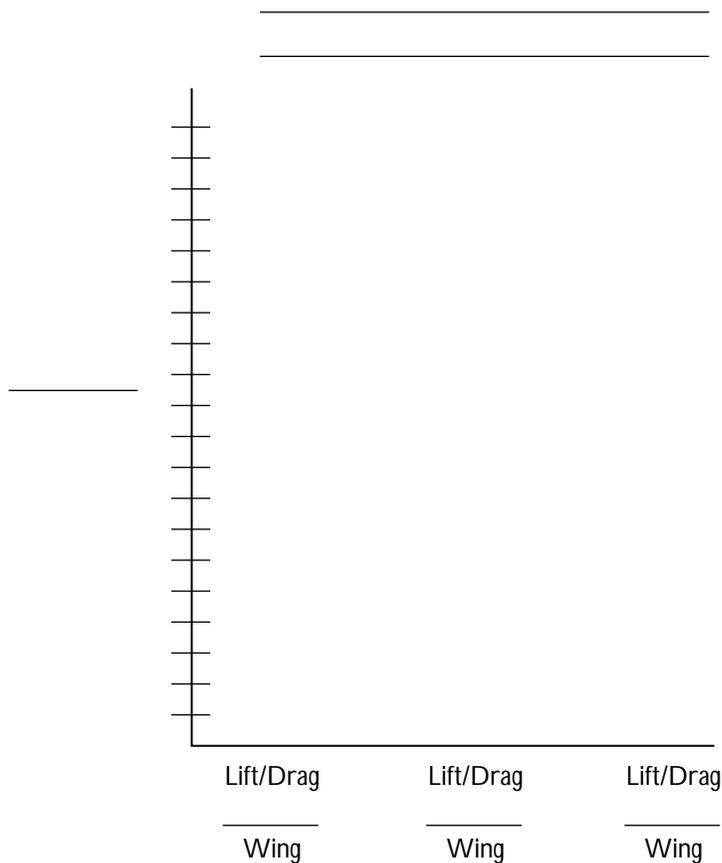
**Example bar graph**  
***X-99 Flight Test Results***





## Exercise 1

**Directions:** Create a bar graph from the averages calculated in the lesson *Wind Tunnel Averages*. Use the template below to create your bar graph. The bar graph should display the average lift and drag for each wing type. Unlike the example bar graph, you will draw two bars for every wing type — one for lift and one for drag.





## Exercise 1 – Key Wind Tunnel Test Results

